SUPERSYMMETRY - A REVIEW

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CERTIFICATE

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Dedicated to my parents

ABSTRACT

In this dissertation we review the basic concepts of supersymmetry-a symmetry between bosons and fermions. In this context we also have briefly reviewed Standard Model (SM) and Grand Unified Theories (GUTs). Supersymmetry algebra and supersymmetric extension of Standard Model has been discussed in a bit detail. We also give a brief account of super GUTs and supergravity (SUGRA).

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INTRODUCTION

In 1970s, particle physics came across a revolution with the experimental confirmation of Glashow-Weinberg-Salam theory of electroweak interactions, which when combined with the Quantum Chromodynamics, is called the Standard Model. Till this day hundreds of experiments have been performed and the experimental data is remarkably in agreement with predictions of this model, up to the energies of the order of 100 GeV or so. But is that all ? What about grand unification and above all unification of gravity with other forces ? What shape does physics take at arbitrarily large energy scales like 10^19 GeV ? These and many other questions suggest that we have to go beyond the standard Model.

There are many theories which are designed to remove problems and inadequacies of the Standard Model like left- right symmetry, grand unified theories, technicolor, compositeness, supersymmetry (SUSY), supergravity, Kaluza- Klein models and superstrings. Which one is favored by nature, experiments will decide, but as far as the mathmatical beauty and conceptual completeness is concerned supersymmetry seems the best.

It has been more than 25 years now since Wess and Zumino [18] initiated a worldwide debate whether there exists a symmetry between bosons and fermions i.e. is nature supersymmetric? Even after 25 years we have not been able to answer this question confidently. But all the theoretical research done so far has revealed the inner beauty and completeness of this theory in such a way that it has made people believe that SUSY must be the symmetry of the nature (even though it is broken at 100 GeV or so).

In this dissertation we have tried to highlight the main ideas of this beautiful theory, and is arranged as follows.

Ch. 1 discusses Standard Model, one of the most beautiful, predictively rich and experimentally varified models given ever. Though neither it includes gravity, nor it addresses many questions of aesthetics yet it is an empirically perfect model for the energies up to the order of 100 GeV where gravity is ignorable.

Ch. 2 deals with Grand Unified Theories (GUTs) : another step towards unification. SU(5) has been considered as an example with predictions of GUTs including Proton decay.

Ch. 3 gives the basic concepts and provides a mathmatical framework for SUSY theories. It might have proved baneficial to discuss/ include the ideas of compositeness and technicolor but since these are not related directly to the core concepts of SUSY so we have just defined them. In Ch. 4 chapter the minimal extension of Standard Model, the MSSM, has been considered in a bit detail.

Last two chapters give brief idea of super GUTs and supergravity theories and finally we give comments and conclusions.

Chapter 1

STANDARD MODEL

1.1 Introduction

Because of its success in describing low energy phenomenology in the number of fundamental fields the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Theory of strong and electroweak interactions with corresponding gauge couplings g_c,g amd g$'$, based on the principle of non-abelian gauge invariance, has become the standard formulation. Where SU(2)$_L$ $\times$ U(1)$_Y$ is the theory of electroweak model[1] and SU(3)$_C$ embodies QCD i.e. the current theory of strong interactions. Here QCD is considered to be an unbroken symmetry of the nature.

In the minimal version, there are two complex Higgs scalars that transform as doublets of SU(2)$_L$ and singlets of SU(3)$_C$. One of the neutral scalars develops a non vanishing vacuum expectation value V, so that the $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory is spontaneously broken at a mass scale of $O(M_w)$ into $SU(3)_L \times U(1)_Q$. Where U(1)$_Q$ is the abelian gauge group of electromagnetism. The three remaining Hermitian Higgs fields are absorbed in a redefinition of gauge bosons providing them with the third degree of freedom. As a consequence, three gauge bosons $(W^{\pm}, Z)$ become massive, while the photon (A_μ) is the gauge boson of the unbroken $U(1)_Q$ group and remains massless. A chiral symmetry forbidding fermion masses is broken at the same time allowing the fermions to become massive. This is a particular example of Higgs Mechanism.

1.2 SM Lagrangian

The SM applies equally to all three families of left-handed doublets of fermions known experimentally:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

with the same set of gauge bosons γ , $W^{\pm}$ and Z, interacting with each family with the same coupling strengths. The right-handed matter fields are SU(2) singlets and they do not participate in weak interactions.

The SM Lagrangian can be written as[2]

$$\begin{aligned} L &= -\frac{1}{4}\vec{W}_{\mu\nu}.\vec{W}^{\mu\nu} - \frac{1}{4}\vec{B}_{\mu\nu}.\vec{B}^{\mu\nu} - \frac{1}{4}\vec{G}^a_{\mu\nu}.\vec{G}^{\mu\nu}_a \\ &+\overline{L}\gamma^\mu D_\mu L + \overline{R}\gamma^\mu D'_\mu R + \overline{L}_q\gamma^\mu D''_\mu L_q \\ &+(D_\mu\phi)^*(D^\mu)^- V(\phi) \\ &-g_1\overline{L}\phi R + g_2\overline{L}\tilde{\phi}R + H.C. \\ &+\frac{1}{2}g_3\overline{\psi}^j_q\gamma^\mu\lambda^a_{jk}\psi^k_q G^a_\mu \end{aligned} \tag{1.1}$$

Where the terms in the first line correspond to $W^{\pm}$, Z^0 , γ and gluon K.E. terms and self interactions. That of the second line to fermionic K.E. and their interactions with $W^{\pm}$, Z^0 and γ.Terms of third line correspond to masses and couplings of $W^{\pm}$, Z^0, γ and Higgs bosons. Whereas terms of fourth line correspond to quark-gluon coupling.

Here the covariant derivatives are defined as

$$
\begin{aligned}
D_\mu &= \left(i\partial_\mu - \frac{1}{2}g\vec{\tau}.\vec{W}_\mu - \frac{1}{2}g'YB_\mu\right), \vec{\tau} = \frac{\vec{\sigma}}{2} \\
D''_\mu &= i\partial_\mu - g_sT_aG^a_\mu - \frac{1}{2}g\vec{\tau}.\vec{W}_\mu \\
D'_\mu &= \left(i\partial_\mu - \frac{1}{2}g'YB_\mu\right)
\end{aligned}
$$

and field strength tensors as

$$
\begin{aligned}
W_{\mu\nu} &= \partial_\mu W^i_\mu - ig'\left[W_\mu, W_\mu\right] \\
G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f_{abc} G^b_\mu G^b_\nu \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu
\end{aligned}
$$

and (W, B) are SU(2) and U(1) gauge fields respectively.

$$
\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \end{pmatrix}, B_\mu
$$

and $\left(\phi, \phi^\dagger\right)$ are scalars with

$$
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}; \phi^\dagger = \left(\phi^-, \overline{\phi^0}\right); \phi^c = i\sigma_2\phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}
$$

i.e. $\phi\left(\widetilde{\phi}\right)$ is Higgs (conjugate) doublet.

After spontaneous symmetry breaking (SSB) the Lagrangian is[3]

$$
L = \sum_i \overline{\Psi}_i(i\partial - m_i\frac{gm_iH}{2M_W})\Psi_i
$$

$$
-\frac{g}{2\sqrt{2}}\sum_i \overline{\Psi}_i\gamma_\mu(1-\gamma^5)(T^+W_\mu^+ + T^-W_\mu^-)\Psi_i
$$

$$
-e\sum_i q_i \overline{\Psi}_i\gamma_\mu\Psi_i A_\mu - \frac{g}{2\cos\theta_w}\sum_i \overline{\Psi}_i\gamma_\mu(g_v^i - g_A^i\gamma^5)\Psi_i Z_\mu \tag{1.2}
$$

where

$$
\theta_w = \tan^{-1}\frac{g'}{g}
$$

is the weak angle;

$$
e = gsin\theta_w
$$

is positron electric charge;

$$
A = Bcos\theta_w + W^3\sin\theta_w
$$

is massless field;

$$
W^\pm = \frac{W^1 \mp iW^2}{\sqrt{2}}
$$

and

$$
Z \equiv -B\sin\theta_w + W^3 cos\theta_w
$$

are massive charged and neutral weak boson field respectively. T^+ , T^- are weak isospin raising and lowering operators. The vectors and axial couplings are

$$
g_v^i \equiv t_{3L}(i) - 2q_i\sin^2\theta_w
$$

$$
g_A^i \equiv t_{3L}(i)
$$

where $t_{3L}(i)$ is the weak isospin of the fermion i and q_i is charge of Ψ_i in units of e. The second term in the L represents the charged current weak interaction. e.g., the coupling of a W to an electron and a neutrino is[4]

$$-\frac{e}{2\sqrt{2}\sin\theta_w}\left[W_\mu^- e^- \gamma_\mu(1-\gamma^5)\nu + W_\mu^+ \overline{\nu}\gamma_\mu(1-\gamma^5)e\right]$$

for $q^2 \ll M_w$ this goes to four-Fermi interaction and comparison gives [4]

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_w^2}$$

In SM lagrangian H is the physical neutral Higgs scalar which is the only remaining part of ϕ after SSB. The mass of Higgs is not predicted by the SM , though experimental limits are given on M_H.

We take e(or α), $\sin\theta_w$ and G_F as input parameter of this model . The prediction of W and Z bosons in terms of these parameters, calculated at tree level [5]

$$M_W^0 = 78.3GeV$$

$$M_Z^0 = 89.0GeV$$

whereas the experimental values are[36]

$$M_W = 80.43GeV$$

$$M_Z = 91.1867GeV$$

The removal of discrepancy requires the need to include the radiative corrections. If we denote the radiative corrections by Δ_r then[6, 7]

$$M_W = \frac{37.28 \pm 0.0003}{\sin\theta_w(1-\triangle_r)^{\frac{1}{2}}} GeV = 83.0 \pm 2.7 GeV$$

and

$$M_Z = \frac{74.562 \pm 0.0006}{\sin 2\theta_w(1-\triangle_r)^{\frac{1}{2}}} GeV = 93.8 \pm 2.2 GeV$$

Thus predicted values are in excellent agreement with the experimental values.

In SM, the CP violation is incorporated by a single observable phase in CKM matrix which relates the quark mass eigen states to weak eigenstates.

1.3 Fermion masses and mixing

Due to the chiral nature of weak interactions, bare mass terms for fermions violate gauge invariance and therefore can not be included in the Lagrangian in order to preserve the renormalizability. Therefore we use the scalar Higgs doublet ϕ to break the gauge symmetries for this purpose and write the gauge invariant Yukawa couplings as[5]

$$\sum_{ij} \left(\overline{L}'_{qi} \Phi d'_{Rj} h^d_{ij} + \overline{L}'_{qi} \tilde{\Phi} u'_{Rj} h^u_{ij} + \overline{L}'_{Li} \Phi e'_{Rj} h^e_{ij} \right) + h.c. \tag{1.3}$$

And subsequent to spontaneous breakdown of gauge symmetry ,these lead to mass terms for fermions. These mass matrices mix the weak eigenstates of different generations and give rise to mixing angles like the Cabibbo angle.

1.4 Successes and inadequacies of SM

The high energy experiments performed in the recent years have proved that SM works with remarkable accuracy. A number of observables measured at the per mil'e level can be successfully fitted in terms of the most relevant parameters of SM i.e. m_t, $\alpha_s(m_z)$ and

m_H. The presence of a few $\sim 2\,\sigma$ deviations is what is to be expected on the statistical grounds. The electroweak data and the SM predicted values obtained from global fit are listed in the table [2], where a remarkable agreement between experiment and theory holds from very good to excellent. But despite all the marvelous achievements of SM up to energies of 100 GeV, there are certain reasons for which SM is thought to be an inadequate model.

1.4.1 Questions and more questions.

i) In SM there are at least 61 particles. (Three generations, each containing 2 leptons and two flavors of tricolored quarks, i.e. 24 fundamental fermion, then their antiparticles, the 12 gauge bosons and one Higgs scalar) can all these really be fundamental?

ii) There are at least 19 arbitrary parameters that are not predicted by the model (nine masses of quarks and charged leptons, three gauge couplings, represented by , four parameters of CKM matrix, the QCD parameters, and the Higgs potential parameters scalar self coupling λ and vacuum expectation value v). This number rises to 26 if neutrinos have mass and there is mixing among the leptons, too.

iii) There is no apparent reason (except CP violation) for the existence of 3 (or more?) generations of quarks and leptons that are almost identical except for their masses.

iv) The SM does not explain why quarks and leptons are so similar in their weak-interaction properties, both occurring in weak isopin doublets.

v) Why the quarks and leptons have charges in such a way that electrons charge -e is equal and opposite to that of the quarks in the proton to at least one part in 10^{21}. If it were otherwise, astronomical sized clumps of matter would mainly interact electrostatically rather than gravitationally? Nor is it clear that why the total charge within each generation vanishes i.e. why

$$3\,(Q_u + Q_d) + Q_{e^-} + Q_\nu = 0$$

which is essential to cancel the SU(2)$\times U(1)$ triangular anomaly.

vi) What is the nature of Higgs bosons and the origin of electro-weak symmetry breaking which is not even experimentally established yet?

vii) The origin of Fermion masses which are much smaller than the scale of electroweak symmetry breaking e.g. $m_{e,u,d} \sim 10^{-5}\Lambda_w$; where Λ_w is the electroweak scale.

viii) Another deficiency of SM is the lack of any physical meaning of the U(1) generator.

ix) What is the origin of parity violation in low energy physics?

1.5 Beyond the Standard Model

From the above discussion it is clear that SM is not complete. Since Higgs is not found yet, there is a possibility of presence of some new physics, what type of physics, SM Higgs, Higgs plus SUSY or new strong forces and Higgs compositeness, this is what experiments will decide. There are also, a few, rare processes that will, if observed, can give hints for new physics which are Neutrino mass, proton decay and neutron-antineutron oscillations and neutrino oscillations. The simplest yet extremely profound example of new physics will be the discovery of non-zero neutrino mass.

Chapter 2

GRAND UNIFIED THEORIES

The SM is not really a unifying theory at all, because there are three different gauge interactions, each with its own coupling strength. Thus it is logical to look for a higher symmetry which unifies all three couplings and possibly can answer some of the unanswered questions of SM.

2.1 Possible choices of Grand Unified Group

The properties which a Grand Unified Group G must possess can be listed as below:

i) As SU(3), SU(2) and U(1) has 2,1,1 diagonal generators, G must have rank $\rangle 4$.

ii) G must have complex representation to incorporate parity violation i.e. $\bar{n} = n^*$ (e.g. in SU(3) $\bar{3} = 3^* \neq 3$).

iii) G should have a single gauge coupling. Thus it should be either a simple group or product of identical simple groups.

iv) Representation of G should accommodate all the known Fermions.

v) It must be free of anomalies in order to be a renormalizable theory.

First attempt in this direction was made by Pati and Salam[11], who unified quarks and leptons within the group $SU(2)_L \times SU(2)_R \times SU(4)_C$ by extending the color gauge group to include the leptons. They explained the quantization of electric charge, although they had three coupling constants g_{2L}, g_{2R} and g_c, since there was no natural left-right symmetry in their model.

The simplest symmetry group which can incorporate the idea of Grand Unification is the $SU(5)$ which corresponds to a rotation in an internal space of 5 dimensions. This model was suggested by Georgi and Glashow [12]. It not only explains the quantization of electric charge but also the unification of all coupling constants let us briefly discuss $SU(5)$.

2.2 SU(5)

In minimal Grand Unification based on $SU(5)$, 15 left handed fermions can be accommodated in $\bar{5}$ and 10 representations of $SU(5)$ with $SU(3)$ and $SU(2)$ content.

$$\overbrace{\underset{d_i^c}{\left(\overline{3},1\right)} + \underset{(\nu,e^-)}{(1,2)}}^{\bar{5}} + \overbrace{\underset{(u_i,d_i)}{(3,2)}, \underset{u_i^c}{\left(\overline{3},1\right)} + \underset{e^c}{(1,1)}}^{10}$$

where for first generation

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu_e \end{pmatrix} ; 10 = \begin{pmatrix} 0 & u_3^c & u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}$$

note that here for each multiplet

$$\sum Q = 0$$

provided

$$Q_d = -\frac{1}{2}Q_u = \frac{1}{3}Q_{e^-}$$

There are 24 gauge bosons associated with $SU(5)$. Under $SU(3) \times \ SU(2)\times \ U(1)$ they decompose as follows

$$\{24\} = \overbrace{(8,1,0) + (1,3,0) + (1,1,0)}_{\text{gluons, } W_i \text{and B fields}} + \overbrace{(3,2,5/3) + (3^*,2,-5/3)}_{\overline{X},\overline{Y},X,Y \text{ fields}}$$

Thus corresponding to 24 generators of $SU(5)$ there are 12 new bosons X, Y,$\overline{X}$,$\overline{Y}$ plus 12 gauge bosons off SM. These new bosons must have charges

$$Q_X = \frac{4}{3};\ Q_Y = \frac{1}{3};$$

X and Y bosons are sometimes called Leptoquarks and they can mediate Proton or Neutron decay. For electroweak theory we defined the Weinberg angle as

$$tan\theta_w = \frac{g_{U(1)}}{g_{SU(2)}} \equiv \frac{g'}{g}$$

now if we take the generator corresponding to Y and take its eigen value for e^+ and then compareing it with $SU(2) \times U(1)$ case we have [5]

$$\sqrt{\frac{3}{5}} g_{SU(5)} = g'$$

but

$$g_{SU(5)} = g'$$

which implies

$$tan\theta_w = \sqrt{\frac{3}{5}}$$

which in turn gives

$$\sin^2\theta_w = \frac{3}{8}$$

This prediction is much larger than experimental value. But when radiative corrections are made we get $\sin^2\theta_w$ =0.21±0.01 which is in remarkable agreement with experiment. In order to break the $SU(5)$ group down to $SU(3)_c \times U(1)_{em}$ two Higgs multiplets are introduced. One belonging to 24-dimensional irreducible representation $\phi(24)$ and the other to 5-dimensional representation $H(5)$ .The stages of symmetry breakdown are given by

$$SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}$$

the first stage is achieved by $< \phi > \neq 0$ and the second stage by $< H > \neq 0$. This spontaneous symmetry breaking gives masses to the gauge bosons.

The predicted value [14] for X,Y boson masses is $\geq 10^{15} GeV$ thus $SU(5)$ symmetry breaks first at about 10^{15} GeV and then at about 10^2 GeV energy scale.

2.3 SO(10)

The other groups that contain the $SU(3)$ color subgroup and that have complex representations are $SU(6)$ and $SO(10)$ both having rank 5.

$SU(6)$ must be excluded since although it has a 15 dimensional representation, its $SU(3) \times SU(2)$ decomposition does not match with our 15 members of one family.

$SO(10)$ contains $SU(5)$ as sub-group and decomposition is

$$16 = 10 + \overline{5} + 1$$

where 10 and $\overline{5}$ are precisely the same as discussed earlier in case of $SU(5)$. For extra $SU(5)$ singlet we can postulate the existence ofν_R.In the minimal $SU(5)$ model neutrino has to be massless so that the observed parity violation seems natural but why it is so is not obvious.

It is perhaps more natural to assume that G possesses L-R symmetry and the parity violation we observe at low energies is the result of symmetry breaking. Thus G must contain $SU(2)_L \times SU(2)_R$ subgroup as in SO(10) and not just $SU(2)_L$ as in SU(5). Thus SO(10) seems more appropriate.

For breaking of SO(10) nature could choose any one of the following ways.

$$SO(10) \rightarrow \begin{array}{c} SU(4) \times SU(2)_L \times SU(2)_R \\ SU(5) \\ SU(4) \times SU(2) \times U(1) \end{array} \rightarrow SU(3) \times SU(2) \times U(1)$$

A nice feature of SO(10) symmetry is that fermions all appear in a single representation and it is automatically free of anomalies. Thus it also satisfies all the requirements for a grand unified theory.

2.4 Hierarchy problem in GUTs

The Hierarchy problem of Grand Unified Theories can be divided into two parts

i) Why $M_w \ll M_{GUT}$ in the first place (more precisely why $M_w \ll M_{planck}$)?

ii) If this Hierarchy is present , how to maintain it ?

The second problem arises due to the fact that there is no known symmetry which can protect scalar masses from quadratic divergences as is the case with fermions and bosons[1].

It is expected that Higgs scalars of electroweak and guts will have masses of the order of M_w and M_{GUT} respectively. But if we take a look on the quantum corrected Higgs

[1]fermion masses are protected by the chiral symmetry and bosons from the gauge symmetries.

mass in SM, it is given by

$$M_H^2 = M_H^2(tree) + \frac{g^2}{16\pi^2}\Lambda^2 + ... \tag{2.1}$$

where g is the weak gauge coupling constant and Λ is the cut-off. Thus due to the presence of cut-off Λ, the quantum corrections to M_H are quadratically divergent. Note that within the context of SM there is no Hierarchy problem because the quadratic divergences can be absorbed in redefinition of M_H in renormalization process. But in case of GUTs , where Higgs couple to the heavy GUT vector bosons, quantum corrections will include terms with $\Lambda \sim M_X$. thus parameters of the theory are to be adjusted in such a way that cancellation at a precision of the order of

$$\frac{M_w^2}{M_H^2} \simeq 10^{-26}$$

is achieved and this requires a "fine tuning". But now another problem arises that every next order in perturbation will destroy this fine tuning so every time we will have to retune our parameters which is a quite unnatural procedure that is why this problem is also called a problem of naturalness. However we will see that SUSY, quite beautifully , deals with this problem. In fact this problem of Hierarchy was one of the basic motivations for SUSY.

2.5 Predictions of GUTs

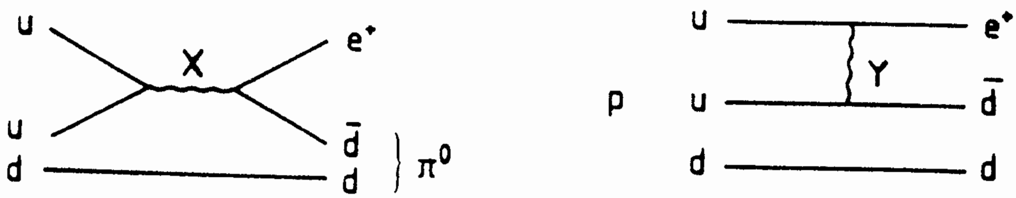
2.5.1 Proton decay

The most spectacular prediction of $SU(5)$ is that proton, which we have hitherto regarded as stable, should decay, Via the exchange of virtual superheavy gauge bosons X and Y.

In $SU(5)$ the most dominant decay of proton is expected to be

$$P \longrightarrow e^+\pi^0$$

Some possible mechanisms for the decay are.



These graphs give rise to proton decay with a life time in minimal version of $SU(5)$

$$\tau\left(P \to e^{+}\pi^{0}\right) \simeq 2 \times 10^{29\pm1.7} years$$

The error includes the uncertainties arising from the measured value of $\Lambda_{\overline{ms}}$[1] and other model-dependent factors. Whereas the present experiment limit for this decay is

$$\tau\left(P \to e^{+}\pi^{0}\right) > 9 \times 10^{32} years$$

which rules out minimal $SU(5)$ as a grand unification gauge group.

2.5.2 Unification of running coupling constants

The grand unification implies that at very high energies all Standard Model couplings became equal i.e.

$$g_s = g' = g_1$$

For $SU(3)_C, SU(2)_L$ and $U(1)_Y$ respectively. As the coupling constants are scale-dependent quantities, therefore these coupling constants must change with energy and if the hypothesis of Grand Unification is to hold they must be equal to $SU(5)$ coupling at some mass M_x. We will call M_u the Grand unification scale. For $\mu < M_x$, $SU(5)$ breaks down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ and separate couplings behave differently. For scale dependence of coupling constants, from Renormalization Group Equations[10], we have

[1]It is QCD scale parameter which roughly denotes the scale at which the chromodynamic interactions become strong. The radiative corrections depend on α_s which in turn depends on $\Lambda_{\overline{MS}}$.

.

$$\beta(\alpha) \equiv \mu \frac{\partial \alpha}{\partial \mu}$$

or with a change of variable ,

$$\beta(\alpha) = -\frac{\partial \alpha}{\partial t}; t = \ln \frac{M_X}{\mu}$$

To one loop order α_s is given by

$$\alpha_s(q^2) = \frac{g_s(q^2)}{4\pi} = \alpha_s^0 \left[1 - \frac{1}{4\pi}\alpha_s^0 b_0 \log\left(\frac{-q^2}{\mu^2}\right)\right]$$

which gives

$$\frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right) = \frac{b_0}{4\pi}$$

integration gives

$$\frac{1}{\alpha} = \frac{b_0}{2\pi}t + c$$

where for Q=μ , c=$\frac{1}{\alpha_i(\mu^2)}$, thus finally to the one loop approximation the solution to RGE gives[10, 5]

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{2\pi}\log\left(\frac{Q}{\mu}\right) \tag{2.2}$$

where

$$\begin{aligned} b_3 &= 11 - \frac{4}{3}N_g \\ b_2 &= \frac{22}{3} - \frac{4}{3}N_g - \frac{1}{6}N_H \end{aligned}$$

$$b_1 = -\frac{4}{3}N_g - \frac{1}{10}N_H$$

where N_g is number of families or generations of fermions and N_H is the number of Higgs doublets in the electroweak sector. Now if we use the known values of strong and electromagnetic coupling along with the above evolution equation, we get the variation of effective coupling constants with energy scale as is shown in Fig 1.

2.5.3 Prediction for magnetic monopoles.

In grand unified theories the electric charge operator is one of the generators of the symmetry and all particles must have the same unit of charge (or fraction of it, as in case of quarks), thus incorporating charge quantization naturally . In 1929 Dirac predicted the existence of magnetic-monopoles with magnetic charge

$$e_m = n\frac{hc}{2e}.$$

In 1974 t′ Hooft [9] and Polyakov showed that magnetic monopoles occur in GUTs naturally with mass $\approx M_X$.

2.5.4 Baryon asymmetry

Guts can provide natural explanation of the baryon asymmetry in the universe. In the universe,

$$\frac{n_B}{n_\gamma} \approx 10^{-9}$$

where, γ' s have typical energies $\approx 10^{-4}$ eV. That is energy of the universe is matter dominant. As GUTs can lead to B through processes such as proton decay. If CP violation is incorporated in GUTs it can be used to calculate $\frac{n_B}{n_\gamma}$ which comes out to be quite close to the observed value .

$$b_1 = -\frac{4}{3}N_g - \frac{1}{10}N_H$$

where N_g is number of families or generations of fermions and N_H is the number of Higgs doublets in the electroweak sector. Now if we use the known values of strong and electromagnetic coupling along with the above evolution equation, we get the variation of effective coupling constants with energy scale as is shown in Fig 1.

2.5.3 Prediction for magnetic monopoles.

In grand unified theories the electric charge operator is one of the generators of the symmetry and all particles must have the same unit of charge (or fraction of it, as in case of quarks), thus incorporating charge quantization naturally . In 1929 Dirac predicted the existence of magnetic-monopoles with magnetic charge

$$e_m = n\frac{hc}{2e}.$$

In 1974 t′ Hooft [9] and Polyakov showed that magnetic monopoles occur in GUTs naturally with mass $\approx M_X$.

2.5.4 Baryon asymmetry

Guts can provide natural explanation of the baryon asymmetry in the universe. In the universe,

$$\frac{n_B}{n_\gamma} \approx 10^{-9}$$

where, γ' s have typical energies $\approx 10^{-4}$ eV. That is energy of the universe is matter dominant. As GUTs can lead to B through processes such as proton decay. If CP violation is incorporated in GUTs it can be used to calculate $\frac{n_B}{n_\gamma}$ which comes out to be quite close to the observed value .

2.6 Successes and problems of GUTs

GUTs have explained many questions unanswered by SM and have also made some aesthetically attractive predictions.

i) The idea of unification of 3 forces is very attractive and aesthetically appealing.

ii) It explains charge quantization.

iii) It explains fractional charges of quarks.

iv) $\sin^2\theta_w$ is correctly predicted.

v) Very small masses of neutrino occur automatically.

vi) B may explain cosmological baryon asymmetry i.e. in universe matter dominates antimatter (although SU(5) is excluded on these bases).

vii) Grand unified theories predict superheavy magnetic-monopoles, that may have been created during phase transitions in early universe and may still exist as relics of those transitions today.

But

i) It does not explain the number of generations of fermions or fermion masses and mixing angles.

ii) Higgs sector is even more arbitrary and complicated than before . larger the group G, more arbitrary is the Higgs sector, greater the number of free parameters and smaller the predictive power.

iii) The Hierarchy of mass scale is another fundamental problem

Chapter 3

WHAT IS NEXT: SUPERSYMMETRY - AN INTRODUCTION

Ultimate unification of all particles and all interactions is the eternal dream of theoretical physicists. The unified gauge theories has taken us a step closer to this goal but they are clearly incomplete. They do not include gravity. They do not explain the number of generations of Fermion masses and mixing angles. In GUTs, Higgs sector is even more arbitrary and complicated than before. And most important and fundamental, the Hierarchy of mass scale.

There has been some attempts to overcome these difficulties. Three most popular solutions proposed are

- **TECHNICOLOR**
- **COMPOSITENESS**
- **SUPERSYMMETRY**

According to **TECHNICOLOR** theory Higgs particles are composite of the so-called techni-fermions F. Here a new confining, asymptotically free, non-Abelian gauge interaction, technicolor, is introduced between these technifermions with a mass scale

$\Lambda_T \approx v \leq 1$ TeV. And according to **COMPOSITE MODELS** all quarks and leptons are composite particles and their hypothetical constituents are usually referred to as "Preons".

Apart from the fact that both these theories introduce new problems and complications, they still solve some of the existing problems and the core ideas are very promising ones, which has been incorporated into a variety of other attempts to unify particle physics beyond the standard model.

The third one, supersymmetry or SUSY, is a symmetry between bosons and fermions. Among all the above proposals SUSY proves to be the most powerful and technically equipped to solve some of the existing problem and paving new ways. Thus we will study this symmetry in a bit more detail.

3.1 Motivation for Supersymmetry:

To see why symmetry between bosons and fermions may be of interest to the study of elementary particle physics we point out that there is no apparent symmetry that can control the divergences associated with scalar field masses. The scalar masses have two sources for their quadratic divergences, one from a scalar loop which comes with a positive sign and another from a fermion loop with negative sign[1]. It is then suggestive that if there was a symmetry that related the couplings and masses of fermions and bosons, all divergences from scalar field masses could be eliminated . Supersymmetry provides such an opportunity. Thus helping to solve the gauge Hierarchy problem. Another motivation to consider SUSY is that it is the most general symmetry of the S-matrix.

One of the most interesting features of SUSY is that the local version of SUSY, SUGRA, is a good candidate for a possible unification of all elementary interactions including Gravity. SUSY appears very naturally in superstring theory which is the current best hope for a "theory of every thing". (However this argument only requires

[1]A well known result of fermi statistics

SUSY at or below Planck scale, not necessarily at the weak scale).Also there are some small hints that SUSY might be true. The most dramatic of these is the unification of couplings.

3.2 SUSY at low energies

There are atleast two reasons for thinking that SUSY might have something to do with nature and that it might be broken at a scale $\approx M_w$

- **Hierarchy problem:** For a theory which contains both massive scalars and fermions in addition to the Higgs field h, we have one loop contribution to M_h^2 is

$$M_h^2 \sim M_{h_0}^2 + \frac{g_F^2}{4\pi^2}\left(\Lambda^2 + m_F^2\right) - \frac{g_S^2}{4\pi^2}\left(\Lambda^2 + m_S^2\right) + \log Divergences \tag{3.1}$$

Now if $g_S = g_F$ we are left with a well behaved contribution to the Higgs boson mass provided the fermion and scalar masses are not too different.

$$M_h^2 \sim M_{h_0}^2 + \frac{g_F^2}{4\pi^2}\left(m_f^2 - m_s^2\right)$$

One can roughly assume that the cancellation is unnatural if the mass spliting is larger than about a TeV. (because $M_h \leq 1$ TeV.

- **SUSY is intrinsic to string theory:**. It is known that there are a vast array of supersymmetric solutions to string theory. A general feature of these solutions is that if SUSY is unbroken in some lowest order the theory remains supersymmetric to all orders. So if string theory describes nature, low energy SUSY is almost certainly a prediction . Since SUSY is by hypothesis relevant to fixing the weak scale of 250 GeV. One expects that difference in masses should not differ by more than 250 GeV. This argument is confirmed in models all of which produce some detectable spartners that are light.

Note that it is not a problem with SUSY ideas that SUSY partners have not yet been found. Most of them are expected to have masses of order M_w and could not have been detected yet. Those that could have been detected, such as gluino, are allowed to be heavy enough to have been unobservable so far.

3.3 SUSY algebra

Symmetries are of fundamental importance in the description of physical phenomena. A fundamental symmetry of particle physics , which has been firmly established both theoretically and experimentally, is that of Poincare′ group, i.e of rotations and translations in four dimensional Minkowski space.

Besides this fundamental symmetry there are other so-called internal symmetries (such as the symmetry of SU(3) flavor group) which has also been firmly established .

Several attempts has been made to unify the space-time symmetry of Poincare′ group with some internal group. But the so called " no-go" theorem of Coleman and Mandula [16]does not allow this.

Coleman Mandula theorem states that if one makes the plausible assumption of locality, causality, positivity of energy, finiteness of number of particles and one more technical assumption then the most general Lie algebra of symmetries of S-matrix ,has the form

$$\begin{aligned} [P_\mu, B_l] &= 0 \\ [M_{\mu\nu}, B_l] &= 0 \end{aligned} \tag{3.2}$$

Where $P_{\mu\nu}$ is the energy momentum operator, $M_{\mu\nu}$ is the Lorentz rotation generator and B_l are a finite number of Lorentz scalar operators. B_l constitutes a Lie algebra

$$[B_l\,,\,B_m] = iC^k_{\rm lm} B_{\rm k} \tag{3.3}$$

Here $C^k_{\rm lm}$ are the structure constants of this Lie algebra of the compact internal symmetry group.

In the simplest words it is not possible to mix space-time symmetry and some internal symmetry except in a trivial way which essentially, does not result in a genuine unification of one group with the other.

The generators of the Poincare$'$ group satisfy well known commutation relations of the field operators which quantize these fields. It was realized by Wess and Zumino [18] that if one allows also the anticommutation relations of generators of a symmetry, called the supersymmetry, then the unification of the space-time symmetry of the Poincare$'$ group with this internal symmetry can be achieved.

The formal proof of this discovery was established by Haag, Lopuszanski and Sohnius[35]. The HLS theorem states that if we generalize the Poincare algebra to a super algebra or graded lie algebra then the maximal symmetry of the S-matrix is the direct product of an internal symmetry with the space-time symmetry. The graded Lie algebra in this case is given by the relations

$$\begin{aligned}
\left\{Q^{\alpha}_{A}, Q^{\beta}_{B}\right\} &= \left\{\overline{Q}^{\alpha}_{\dot{A}}, \overline{Q}^{\beta}_{\dot{B}}\right\} = 0 \\
\left\{Q^{\alpha}_{A}, \overline{Q}^{\beta}_{\dot{B}}\right\} &= 2\delta^{\alpha\beta}\sigma^{\mu}_{A\dot{B}}P^{\mu} \\
\left\{Q^{\alpha}_{A}, P_{\mu}\right\} &= \left\{Q^{\alpha}_{\dot{A}}, P_{\mu}\right\} = 0 \\
\left\{Q^{\alpha}_{\dot{A}}, B_{l}\right\} &= iS^{\alpha\beta}_{l}Q^{\beta}_{A} \\
\left[Q^{\alpha}_{\dot{A}}, M^{\mu\nu}\right] &= i\left(\sigma^{\mu\nu}_{2}\right)^{B}_{A}Q^{\alpha}_{B} \\
\left[B_{l}, B_{m}\right] &= iC^{k}_{lm}B_{k}
\end{aligned} \tag{3.4}$$

where Q_A ,$\overline{Q}_{\dot{A}}$ are the 2-component Weyl spinors (and indices A, B=1,2) which are added to Poincaré algebra to extend it to the superalgebra. $S^{\alpha\beta}_{l}$ are the Hermitian representation matrices of the representation. containing the charges. Q^{α}_{A} and B_l are the generators of the internal symmetry group.

The only allowed extension is the possible appearance of so-called central charges in the anticommutator of two undotted spinors. Thus instead of the first relation of (1) one can have,

$$\left\{Q_A^\alpha, Q_B^\beta\right\} = \epsilon_{AB} Z^{\alpha\beta} \tag{3.5}$$

where

$$\epsilon_{AB} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \left(\epsilon^{AB}\right)^{-1}, Z^{\alpha\beta} = -Z^{\beta\alpha}$$

also

$$\left[Z^{\alpha\beta}, B_l\right] = 0$$

which is the reason why the quantities Z are called central charges.

Consider two pairs of creation and annihilation operators (b,$b^\dagger$) and (f,$f^\dagger$) for bosons and fermions satisfying the following relations,

$$\left[b, b^\dagger\right] = 1$$

$$\left\{f, f^\dagger\right\} = 1$$

The Hamiltonian for this system can in general be written as

$$H = W_b b^\dagger b + W_f f^\dagger f$$

Now if we define a fermionic operator,

$$Q = b^{\dagger f} + f^\dagger b$$

then we have

$$\begin{aligned}
\left[Q, b^{\dagger}\right] &= f^{\dagger} \\
\left\{Q, f^{\dagger}\right\} &= b^{\dagger}
\end{aligned}$$

Thus if $f^{\dagger} \mid 0\rangle$ and $b^{\dagger} \mid 0\rangle$ represent fermionic and bosonic states respectively, Q will take bosons to fermions and vice-versa. also

$$[Q, H] = (\omega_b - \omega_f)\, Q$$

So for equal energies for bosonic and fermionic states H is supersymmetric. Also for this case

$$[Q, Q^{\dagger}]_{+} = \frac{2}{\omega} H.$$

Thus algebra of $Q, Q^{\dagger}$ and H closes under anticommutation.

For more than one f and b there must be equal number of them otherwise last two equations can not be satisfied together. This is a simple illustration that SUSY requires equal number of fermionic and bosonic degrees of freedom and that anticommutators of Q and $Q^{\dagger}$ involve Hamiltonian which is an important implication for physics.

3.3.1 Generators of SUSY

It is quite clear that symmetry we are looking for, must connect bosons and fermions which in turn implies that the generators themselves carry half-integer spin i.e. are fermionic in nature. Whereas the generators of Lorentz group or that of the gauge group are all bosonic in nature.

As is clear from HLS theorem, for presentation of SUSY we do not change the structure of space time but we add structure to it. We start with usual four coordinates $x^{\mu} = (t, x, y, z)$ and add four odd dimensions $Q_{\alpha}(\alpha = 1, ..., 4)$. These odd dimensions are fermionic and anticommute. Thus SUSY is formulated as a generalization of space-time symmetry to include spinorial generators which obey specific anticommutation relations e.g. the Lie algebra of SU(2) is given by eqn.

$$[J_i, J_j] = i\epsilon_{ijk} J_k$$

whereJ_i are generators of SU(2) under a rotation described by the angles $\overrightarrow{\theta}$ $(\theta_1, \theta_2, \theta_3)$. The field components mix according to

$$\Psi \rightarrow e^{i\overrightarrow{\theta}\cdot\overrightarrow{J}} \Psi$$

then for two infinitesimal rotations $\delta_1\Psi$ and $\delta_2\Psi$ it is easy to check that

$$[\delta_1, \delta_2] = i\epsilon_{ijk}\theta_i^1\theta_j^2 J_k$$

in the same way if we take a supersymmetric Lagrangian like

$$L = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{1}{4}\overline{\Psi} i\gamma^\mu\partial_\mu\Psi \tag{3.6}$$

Where L is effectively invariant under transformations which mix the scalar fields A, B and spinor field Ψ, then we can see that in this case

$$[\delta_1, \delta_2] = (2\overline{\epsilon_2}\gamma^\mu\epsilon_1) P_\mu.$$

The similarity of this equation with operator equation for rotations suggests that whatever the generators of SUSY are, the generalized algebra which they obey will be expected to involve the generator P_μ of Poincaré group i.e. SUSY transformations do not form a group but are just an extension of the Poincaré group. Using the example of SU(2) again we introduce generators of SUSY transformations Q, in analogy with

$$\delta\Psi = i\theta_n J_n \Psi$$

whereθ_n are parameter of transformation. we have

$$\delta\,(Field) = i\epsilon^A Q_A\,(Field)$$

here ϵ^A are the parameters of transformation and Q_A are the generators. They are both Grassmann quantities. in Weyl 2-spinor representation we write,

$$\epsilon^A = \begin{pmatrix} \eta^\alpha \\ \overline{\eta}_{\beta^\cdot} \end{pmatrix}, Q_A = \begin{pmatrix} Q_\alpha \\ \overline{Q}_{\beta^\cdot} \end{pmatrix}$$

Since spinors are intrinsically complex objects so $Q^\dagger$ is also a symmetry generator. $Q, Q^\dagger$ are Fermionic operators they carry spin angular-momentum 1/2. it again makes it clear that SUSY must be a space time symmetry and LHS theorem implies that these generators must satisfy an algebra of anticommutation and commutation relation with Schematic form

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= \left\{\overline{Q}_{\alpha^\cdot}, \overline{Q}_{\beta^\cdot}\right\} = 0 \\
\left\{Q_\alpha, \overline{Q}_{\beta^\cdot}\right\} &= 2\sigma^\mu_{\alpha\beta^\cdot} P^\mu \\
[Q_\alpha, P_\mu] &= = 0
\end{aligned}$$

Here we have chosen the simplest choice of SUSY generators which is a 2-component Weyl spinor Q and its conjugate $\overline{Q}_{.}$ where $\alpha, \beta, \overline{\alpha}^\cdot, \overline{\beta}^\cdot$ take values 1,2.

3.3.2 Superspace

A finite supertranslation or SUSY transformation is defined as

$$S(x, \theta, \overline{\theta}) = e^{i(\theta^\alpha Q_\alpha + \overline{\theta}_{\alpha^\cdot} \overline{Q}^{\alpha^\cdot} - X^\mu P_\mu)} \tag{3.7}$$

whereθ^{α} and $\overline{\theta}_{\alpha^{\cdot}}$ and are anticommuting Grassmann variables and are 2-component spinors obeying

$$\{\theta, \theta\} = \left\{\theta, \overline{\theta}\right\} = \left\{\overline{\theta}, \overline{\theta}\right\} = 0$$

Then it is clear that objects or fields on which these SUSY transformations act must also depend onθ and $\overline{\theta}$. This leads to the introduction of superfield and superspace. The superspace is defined as

$$Z = (x^{\mu}, \theta^{\alpha}, \overline{\theta}_{\alpha^{\cdot}})$$

with four fermionic and four bosonic coordinates. For ordinary translations we have

$$e^{i(X.P)}.e^{i(Y.P)} = e^{i(X+Y).P}, [P_{\mu}, P_{\nu}] = 0$$

If we try to combine two supertranslations we have

$$e^{i(a.p+\xi Q+\overline{\xi Q})}.e^{i(x.p+\theta Q+\overline{\theta Q})} = e^{i(x'.p+\theta' Q+\overline{\theta' Q})}$$

Using Hausdorff formula and the relations of graded Lie algebra we and then comparing left and right hand sides we have

$$\begin{aligned}
x^{\mu} &= (a+x)^{\mu} + i\left(\xi\overline{\sigma}\overline{\theta} - \theta\sigma\overline{\xi}\right) \\
\theta'_{\alpha} &= \xi_{\alpha} + \theta_{\alpha} \\
\overline{\theta}'_{\alpha^{\cdot}} &= \overline{\xi}_{\alpha^{\cdot}} + \overline{\theta}_{\alpha^{\cdot}}
\end{aligned}$$

This shows that translation is space-time coordinates involve translation in fermionic coordinates which implies that we can not take

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\mu}$$

In analogy with

$$P_\mu = -i\frac{\partial}{\partial x^\mu}$$

So we define Q_α as

$$\begin{aligned} iQ_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\alpha\cdot}\overline{\theta}^{\alpha\cdot}\partial_\mu i \\ \overline{Q}^{\alpha\cdot} &= \frac{\partial}{\partial\overline{\theta}_{\alpha\cdot}} - i\theta^\alpha\sigma^{\mu\alpha\cdot}_\alpha\partial_\mu \end{aligned}$$

These definitions are compatible with our graded Lie algebra i.e. they satisfy

$$\left\{Q_\alpha, \overline{Q}_{\beta\cdot}\right\} = 2\sigma^\mu_{\alpha\beta\cdot}P^\mu etc.$$

3.3.3 Superfields

The concept of superfields was first proposed by Salam and Strethdee [37]. They proposed that a function of superspace coordinates can generate the components of the supermultiplets.

Consider a function of superspace

$$\Phi = \Phi(x, \theta, \overline{\theta}^\cdot)$$

A Tailor expansion in the variables θ and $\overline{\theta}$ is given by

$$\Phi(x,\theta,\overline{\theta}) = C(x) + \theta\phi(x) + \overline{\theta}\overline{\chi}(x) + \theta\theta M(x) + \overline{\theta\theta}N(x) + \theta\sigma^{\mu}\overline{\theta}V_{\mu}(x)$$
$$+\theta\theta\overline{\theta\lambda}(x) + \overline{\theta\theta}\theta\psi(x) + \theta\theta\overline{\theta\theta}D(x) \quad (3.8)$$

HereC(x), M(x), N(x), $\phi(x)$, $\overline{\chi}(x)$, $\overline{\lambda}(x)$, $\psi(x)$, $V_{\mu}(x)$, d(x)are component fields with 16 bosonic and 16 fermionic degrees of freedom.i.e. a superfield in 8-dimensions is equivelent to 16 -component set of ordinary fields in 4-dimensions.

The covariant derivatives[1] for the superfields are defined as

$$\begin{aligned} D_A &\equiv \partial_A + i\sigma^{\mu}_{A\dot{B}}\overline{\theta}^{\dot{B}}\partial_{\mu} \\ \overline{D}_{\dot{A}} &\equiv -\overline{\partial}_{\dot{A}} - i\theta^{B}\sigma^{\mu}_{B\dot{A}}\partial_{\mu} \end{aligned} \quad (3.9)$$

These operators obey the following relations[2]

$$\{D,D\} = \left\{\overline{D},\overline{D}\right\} = 0 \quad (3.10)$$

$$\{D,Q\} = \left\{\overline{D},\overline{Q}\right\} = \left\{\overline{D},Q\right\} = \left\{D,\overline{Q}\right\} = 0 \quad (3.11)$$

and

$$D^3 = \overline{D}^3 = 0 \quad (3.12)$$

$$\left\{D_A,\overline{D}_{\dot{B}}\right\} = 2\sigma^{\mu}_{A\dot{B}}P_{\mu} \quad (3.13)$$

These covariant derivatives can be used to define the following projection operators,

[1]i.e. the operators which are invariant under supersymmetric transformations

[2]For the detailed mathmatical proofs of these relations see[]

$$\begin{aligned}
\pi_+ &\equiv -\frac{1}{16\Box}\overline{D}^2 D^2 && (3.14)\\
\pi_- &\equiv -\frac{1}{16\Box}D^2\overline{D}^2 && (3.15)\\
\pi_T &\equiv \frac{1}{8\Box}D^A\overline{D}^2 D_A && (3.16)
\end{aligned}$$

Now it is clear from the above definitions that

$$\begin{aligned}
\overline{D}\pi_+ &= 0 \qquad as\overline{D}^3 = 0 && (3.17)\\
D\pi_- &= 0 \qquad asD^3 = 0 && (3.18)
\end{aligned}$$

Thus if we define superfields

$$\phi_\mp \equiv \pi_\pm \Phi \tag{3.19}$$

Then

$$\begin{aligned}
\overline{D}\phi_- &= 0 && (3.20)\\
D\phi_+ &= 0 && (3.21)
\end{aligned}$$

and the fields obeying these constraintsare called left handed and right handed **chiral superfields** respectively.i.e

$$\begin{aligned}
\overline{D}\Phi_L &\equiv 0\\
D\Phi_R &\equiv 0
\end{aligned}$$

There is another kind of superfields, **vector superfields** which obey the reality condition i.e.

$$\Phi(x,\theta,\overline{\theta}) = \Phi^{\dagger}(x,\theta,\overline{\theta})$$

Now applying this condition on the expansion of field $\Phi(x,\theta,\overline{\theta})$ we get

$$\begin{aligned} C(x) &= C^{*}(x), \phi(x) = \chi(x), M(x) = N^{*}(x) \\ V_{\mu}(x) &= V*_{\mu}(x)\ , \lambda(x) = \psi(x), D(x) = D^{*}(x) \end{aligned}$$

Hence a vector superfield obeying these conditions has a general expansion of the form.

$$\begin{aligned} V(x,\theta,\overline{\theta}^{\cdot}) &= C(x) + \theta\phi(x) + \overline{\theta\phi}(x) + \theta\theta M(x) + \overline{\theta\theta}M^{*}(x) + \theta\sigma^{\mu}\overline{\theta}V_{\mu}(x) \\ &+\theta\theta\overline{\theta\lambda}(x) + \overline{\theta\theta}\theta\lambda(x) + \theta\theta\overline{\theta\theta}D(x) \end{aligned} \tag{3.22}$$

Where $C(x), V_{\mu}(x)$ and $D(x)$ are real fields. $M(x), D(x)\,\text{and}\,C(x)$ are scalar fields. $\lambda(x), \phi(x)$ are spinor fields and $V_{\mu}(x)$ is a vector field.

The **supersymmetric field strength** for an arbitrary vector superfield $V(x,\theta,\overline{\theta}^{\cdot})$ is defined by the components

$$W_{A} \equiv -\frac{1}{4}\left(\overline{DD}\right)D_{A}V(x,\theta,\overline{\theta}^{\cdot}) \tag{3.23}$$

$$\overline{W}_{\dot{A}} \equiv -\frac{1}{4}(DD)\overline{D}_{\dot{A}}V(x,\theta,\overline{\theta}^{\cdot}) \tag{3.24}$$

where W_{A} and $\overline{W}_{\dot{A}}$ are examples of spinor superfields.

A general non-abelian SUSY gauge transformation acting on V can be described by

$$e^{gV} \rightarrow e^{-ig\Lambda^{\dagger}} e^{gV} e^{ig\Lambda}$$

where $\Lambda(x, \theta, \overline{\theta})$ is a chiral superfield and g is the gauge coupling.

3.4 Wess Zumino model

The simplest way to display the supersymmetry is to write a Lagrangian containing fermion and bosons fields i.e

$$L = \frac{1}{2} (\partial_\mu A)^2 + \frac{1}{2} (\partial_\mu B)^2 \frac{1}{4} \overline{\Psi} i \gamma_\mu \overleftrightarrow{\partial} \Psi \tag{3.25}$$

where A and B are scalar fields and Ψ is a Majorana spinor field. This is simplified Wess Zumino Model[]. This Lagrangian is effectively invariant (up to a total derivative) under transformations which mix the scalar and spinor fields. These supersymmetric transformations are

$$\begin{aligned} \delta A &= \bar{\epsilon} \Psi \\ \delta B &= i \bar{\epsilon} \gamma^5 \Psi \\ \delta \Psi &= -i \gamma^\mu \epsilon (\partial_\mu A) + \gamma^\mu \gamma^5 \epsilon (\partial_\mu B) \\ \delta \overline{\Psi} &= i \bar{\epsilon} \gamma^\mu (\partial_\mu A) - \bar{\epsilon} \gamma^5 \gamma^\mu (\partial_\mu B) \end{aligned}$$

Now let $\delta_{1,}\delta_2$ be relevant field variations corresponding to the Majorana spinors ϵ_1 and ϵ_2 respectively then

$$[\delta_{1,} \delta_2] A = \bar{\epsilon}_2 (\delta_1 \Psi) - \bar{\epsilon}_1 (\delta_2 \Psi)$$

$$
\begin{aligned}
&= -2i\bar{\epsilon}_2\gamma^\mu\epsilon_1\left(\partial_\mu A\right) \\
\left[\delta_1,\delta_2\right] A &= 2\bar{\epsilon}_2\gamma^\mu\epsilon_1 P_\mu A \\
\left[\delta_1,\delta_2\right] B &= 2\bar{\epsilon}_2\gamma^\mu\epsilon_1 P_\mu B \\
\left[\delta_1,\delta_2\right] \Psi &= 2\bar{\epsilon}_2\gamma^\mu\epsilon_1\Psi - \bar{\epsilon}_2\gamma_\rho\epsilon_1\gamma^\mu\gamma^\rho P_\mu\Psi
\end{aligned}
$$

where to get the last term we have made use of Fierz rearrangement formula along with other properties of Majorana spinors. Note that first term on R.H.S. of eq. is representing a translation as in case of first two eqns. and this is what was expected but second term is undesirable. To eliminate this term we enlarge the definition of $\delta\Psi$ and include fields F and D and take

$$\delta\Psi = -i\gamma^\mu\epsilon\left(\partial_\mu A\right) + \gamma^\mu\gamma^5\epsilon\left(\partial_\mu B\right) - \epsilon F - i\gamma^5\epsilon D$$

where F and D now have additional relations

$$
\begin{aligned}
\delta F &= i\bar{\epsilon}\gamma^\mu\left(\partial_\mu\Psi\right) \\
\delta D &= -\bar{\epsilon}\gamma^5\gamma^\mu\left(\partial_\mu\Psi\right)
\end{aligned}
$$

It is very easy to check that $[\delta_1,\delta_2]\, A$ and $[\delta_1,\delta_2]$B are unchanged with these transformations and

$$\left[\delta_1,\delta_2\right]\Psi = 2\bar{\epsilon}_2\gamma^\mu\epsilon_1\ P_\mu\Psi$$

with

$$
\begin{aligned}
\left[\delta_1,\delta_2\right] F &= 2\bar{\epsilon}_2\gamma^\mu\epsilon_1 P_\mu F \\
\left[\delta_1,\delta_2\right] D &= 2\bar{\epsilon}_2\gamma^\mu\epsilon_1 P_\mu D
\end{aligned}
$$

We have got the desired result but now the problem is that Lagrangian is no longer su-persymmetric, unless we add term $\frac{1}{2}F^2 + \frac{1}{2}D^2$ in the Lagrangian so that $\delta\left(\frac{1}{4}\overline{\Psi}\ldots\right)$ is cancelled by $\delta\left(\frac{1}{2}F^2 + \frac{1}{2}D^2\right)$ and the Lagrangian is invariant apart from a total diver-gence. Thus finally we have the action

$$S = \int \left[\frac{1}{2}\left(\partial_\mu A\right)^2 + \frac{1}{2}\left(\partial_\mu B\right)^2 \frac{1}{4}\overline{\Psi} i\gamma_\mu \overleftrightarrow{\partial} \Psi + \frac{1}{2}F^2 + \frac{1}{2}D^2\right]$$

which is invariant under supersymmetric transformations

$$\begin{aligned}
\delta A &= \bar{\epsilon}\Psi \\
\delta B &= i\bar{\epsilon}\gamma^5\Psi \\
\delta F &= i\bar{\epsilon}\gamma^\mu\left(\partial_\mu\Psi\right) \\
\delta D &= -\bar{\epsilon}\gamma^5\gamma^\mu\left(\partial_\mu\Psi\right) \\
\delta\Psi &= -i\gamma^\mu\epsilon\left(\partial_\mu A\right) + \gamma^\mu\gamma^5\epsilon\left(\partial_\mu B\right) - \epsilon F - i\gamma^5\epsilon D \\
\delta\overline{\Psi} &= i\bar{\epsilon}\gamma^\mu\left(\partial_\mu A\right) - \bar{\epsilon}\gamma^5\gamma^\mu\left(\partial_\mu B\right) - \bar{\epsilon}F - i\bar{\epsilon}\gamma^5 D
\end{aligned}$$

Note that the number of bosonic degrees of freedom (*i.e.*4) is equal to the number of fermionic degrees of freedom and this is the characteristic of SUSY theories.

3.5 Non-renormalization of SUSY theories

One of the most beautiful aspects of these theories is that here the quadratic divergences contributing from fermionic and bosonic loops are cancelled leaving a finite theory. Con-sider a supersymmetric lagrangian,

$$L = L_0 + L_{int}$$

where

$$
\begin{aligned}
L_0 &= \frac{1}{2}\left(\partial_\mu A\right)^2 + \frac{1}{2}\left(\partial_\mu B\right)^2 \frac{1}{4}\overline{\Psi} i\gamma_\mu \overleftrightarrow{\partial} \Psi \\
&+ \frac{1}{2}F^2 + \frac{1}{2}D^2 + mDA + mFB
\end{aligned} \tag{3.28}
$$

$$
\begin{aligned}
L_{int} &= -\frac{g}{\sqrt{2}} A\overline{\Psi}\Psi + i\frac{g}{\sqrt{2}} B\overline{\Psi}\gamma_5\Psi + \\
&\frac{g}{\sqrt{2}}(A^{2-}B^2)D + g\sqrt{2}ABF
\end{aligned} \tag{3.29}
$$

now Euler-Lagrange equations for F and D are

$$
\begin{aligned}
F &= -mB - g\sqrt{2}AB \\
D &= -mA - g\left(A^2 + B^2\right)
\end{aligned}
$$

substituting back into the Lagrangian we get

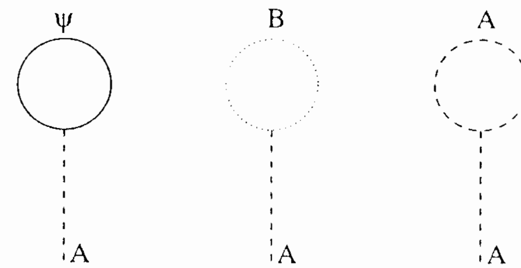
$$
\begin{aligned}
L = & \frac{1}{2}\left(\partial_\mu A\right)^2 + \frac{1}{2}\left(\partial_\mu B\right)^2 \frac{1}{4}\overline{\Psi} i\gamma_\mu \overleftrightarrow{\partial} \Psi - \frac{1}{2}m^2\left(A^2 + B^2\right) - \frac{1}{2}m\overline{\Psi}\Psi \\
& -\frac{g}{\sqrt{2}}A\overline{\Psi}\Psi + i\frac{g}{\sqrt{2}}B\overline{\Psi}\gamma_5\Psi - g\sqrt{2}AB^2 - \frac{gm}{\sqrt{2}}A(A^{2-}B^2) - g^2A^2B^2 \\
& -\frac{g^2}{4}(A^{2-}B^2)
\end{aligned}
$$

here the generating functional for field A,B and Ψ of Wess-Zumino Lagrangian is

$$
\begin{aligned}
Z_L\left[g, J, \eta, \overline{\eta}\right] &= \left[e^{i\int L_{int}}\right] Z_0\left[g, J, \eta, \overline{\eta}\right] \\
Z_0\left[g, J, \eta, \overline{\eta}\right] &= Ne^{\left[\,\frac{i}{2}\int J(x)\Delta_F(x-y)J(y)d^4xd^4y - \frac{i}{2}\int g(a)\Delta_F(a-b)g(b)d^4ad^4b - i\int \overline{\eta}S\eta\right]}
\end{aligned}
$$

Using this we can determine the Greens functions for each of the interaction terms in Wess Zumino model and then can calculate one point functions for fields A,B and ψ.The one point functions of A in lowest orders of perturbation [calculated in detail in] are

$$\frac{g}{\sqrt{2}}\int[-i\Delta_{K_{43}}]-[iS_{33}]\,d_3$$
$$-\frac{gm}{\sqrt{2}}\int[-i\Delta_{K_{43}}][-i\Delta_{P_{33}}]\,d_3$$
$$-\frac{gm}{\sqrt{2}}\int[-i\Delta_{K_{33}}][-i\Delta_{K_{43}}]\,d_3$$



thus we have got the Feynman diagrams and multiplicative factors so applying Feynman rules [20]we can write

$$\begin{aligned}\langle 0 \mid L_{int} \mid A\rangle &\sim \frac{g}{\sqrt{2}}\left[Tr\int\frac{d^4p}{p-m_\Psi}-m\int\frac{d^4p}{p^2-m_B^2}-3m\int\frac{d^4p}{p^2-m_A^2}\right]\\ &= \frac{g}{\sqrt{2}}\left[4m_\Psi\int\frac{d^4p}{p^2-m_\Psi^2}-m\int\frac{d^4p}{p^2-m_B^2}-3m\int\frac{d^4p}{p^2-m_A^2}\right] \qquad (3.30)\end{aligned}$$

In a supersymmetric theory as all the masses are exactly equal all the three contributions add up to zero. Thus although each diagram is separately quadratically divergent but the divergence from fermion loop exactly cancels the sum of divergences from boson loops.

It should be noted that for cancellation to occur M_Ψ should be equal to mass m that enters via the trilinear scalar interactions and if M_Ψ is different from bosonic masses the above expression is logarithmically divergent. And this allows m_s to be different from m_F.

3.6 Supersymmetry breaking

By definition, in the limit of exact supersymmetry fermions and bosons are degenerate in mass. But there is no Selectron with .511 MeV mass or Smuon with mass .106

MeV. In fact not any superpartner have been discovered yet. Thus in order to relate supersymmetry with real world we must consider models where SUSY is broken. This breaking could be either explicit or spontaneous.

We will preferably consider the **spontaneous breaking** of the symmetry which has already been successfully applied for the breaking of gauge symmetries. But unfortunately it is not easy to break SUSY spontaneously because

i) If we introduce a spin 0 field with negative mass-squared, its fermionic spartner would have an imaginary mass .

ii) From SUSY algebra we have

$$\left\{Q_\alpha, Q_\delta^\dagger\right\}\gamma^0_{\delta\beta} = 2\gamma^\mu_{\alpha\beta}P_\mu$$

which for $\mu = 0$ and $\alpha = \beta$ gives

$$\sum_\alpha \left\{Q_\alpha, Q_\alpha^\dagger\right\} = 8P_0 = 8H$$

and hence

$$\langle 0 \mid H \mid 0\rangle = \sum_\alpha |Q_\alpha \mid 0\rangle|^2 + \sum_\alpha \left|Q_\alpha^\dagger \mid 0\rangle\right|^2$$

that is the vacuum energy is positive definite. So if SUSY breaks spontaneously $E_{vac}\rangle 0$ this might give rise to a trouble some cosmological constant.

iii) In the spontaneous breaking of SUSY (in the absence of gravity),one would expect spin 1/2, massless Goldstino (G) which is excluded experimentally. This G would then be the LSP. However in SUGRA models the G is absorbed by the gravition ($g_{3/2}$) which then acquires a mass.

In case of **explicit SUSY breaking** (by adding extra terms in the Lagrangian which will break supersymmetry of the Lagrangian) the arbitrariness in the theory increases, thereby reducing its predictive power. In addition we would inevitably lose the non-renormalization theorems and even worse ,any attempt to include gravity via local SUSY would be prohibited.

Chapter 4

MSSM: THE SUPERSYMMETRIC EXTENSION OF THE STANDARD MODEL

4.1 Introduction

At present there exists a successful (at low energies) model of electro- weak and strong interactions - the Standard model. Let us consider the possibility that new physics beyond may be related to supersymmetry i.e. we take the supersymmetric extension of the SM, The **M**inimal **S**upersymmetric **St**andard **M**odel. The MSSM respects the same gauge symmetries as does the SM. For its supersymmetric extension we must double the entire particle spectrum, placing the observed particles in superfields with new postulated superpartners. The superfield of MSSM are shown in the table (3). Note that each quark has two scalar partners, one corresponding to each quark chirality. The leptons are contained in the doublet superfield which in itself contains the left handed fermions and their scalar partners sleptons. The gauge fields all obtain Majorana fermion partners in a SUSY model.

. Here it should be noted that the subscript of "t" and "R" in squarks and s leptons does not correspond to chirality. (because they are scalars) but they are written just to recognize that they belong to left handed quarks and leptons or to right handed ones. Also we see that gauge fields all obtain Majorana Fermion partners in a SUSY model.

There is one "unusual" thing here, that is the presence of two Higgs doublets which then obtain 2 more Higgs doublets in SUSY extension. The reason is that if we start with the one Higgs doublet as is the case in SM it will acquire a SUSY partner which is a doublet of Majorana fermion fields, (Higgsinos) which contribute to the triangle ... gauge an amolies. As all other anomalies of the model are cancelled among themselves it follows that contribution from fermionic partner of the Higgs doublet remains uncancelled. To make the theory sensible, these anomalies must be cancelled somehow. The simplest way is to add a second Higgs doublet with precisely the opposite quantum numbers. In this way MSSM is infect SUSY extension of extended Standard Model with two Higgs doublets

In a SUSY model this second Higgs doublet will also have fermionic partner. ... contributions of the fermionic partners of the two Higgs doublets to gauge anomalies ... precisely cancel each other leaving an anomaly free theory.

Later we will see that two Higgs doublets are also required in order to give ... the up and down quarks masses in a SUSY theory. From the table it is easy to check that matter fermions fields satisfy the condition for anomaly cancellation. Note that superfields contain "auxiliary fields" also. These are fields with no K.E. terms in the Lagrangian

4.2 MSSM Lagrangian

While constructing the MSSM Lagrangian the important points to be taken care of, apart from those necessary to construct SM Lagrangian (e.g. gauge invariance), are

i) we have to include the superpartners of all the particles in SM with two Higgs doublets.

ii) SUSY invariance.

iii) Inclusion of "Soft" breaking terms for SUSY breaking.

iv) An exact discrete R-parity.

It is seen that if we construct a theory based on all the points except the last one, we get baryon-and lepton-number violating terms in the Lagrangian, which lead to unacceptable physics (fast rates of nucleon decay). It is believed that these terms can only be avoided in a satisfactory way by introducing some additional symmetries. There are various possibilities one can imagine. However, in MSSM we use the unbroken R-symmetry. The R parity of a state is related to its spin S, baryon number B and lepton number L according to

$$R_p = (-1)^{2J+3B+L}$$

All SM particles are R-even and their superpartners are R-odd. More will be said on R-parity later. In general the complete MSSM Lagrangian will have a form

$$L = L_{SUSY} + L_{SOFT}$$

here first part is supersymmetric and the second part explicitly breaks SUSY. The term L_{SUSY} is obtained by "supersymmetrizing" the ordinary SM Lagrangian.

We will consider leptons only. gauge invariant coupling of $\Phi$and V is

$$L_\Phi = \Phi^\dagger e^{gv} \Phi$$

This gives rise to the gauge coupling of the matter fields after we expand the exponential and note:

$$L_\Phi = \Phi^\dagger \Phi + g\Phi^\dagger V \Phi + \frac{1}{2} g \Phi^\dagger V^2 \Phi.$$

Thus we can write[1]

$$
\begin{aligned}
L_{L,q} &= \int d^4\theta \left[\hat{L}^{\dagger} e^{2gV + g'\hat{B}} \hat{L} + \hat{R}^{\dagger} e^{2gV + g'\hat{B}} \hat{R} \right] \\
&+ \int d^4\theta \left[\hat{Q}^{\dagger} e^{2gV + g'\hat{B}} \hat{Q} + \hat{u}^{\dagger} e^{2gV + g'\hat{B}} \hat{u} \right] \\
&+ \int d^4\theta \left[\hat{Q}^{\dagger} e^{2gV + g'\hat{B}} \hat{Q} + \hat{u}^{\dagger} e^{2gV + g'\hat{B}} \hat{u} \right]
\end{aligned}
\tag{4.1}
$$

$$
L_{guage} = \frac{1}{4} \int d^4\theta \left[W^{a\alpha} W^a_\alpha + W'^{\alpha} W'_\alpha \right] \delta^2 \left(\bar{\theta} \right) + h.c. \tag{4.2}
$$

and

$$
\begin{aligned}
L_{Higgs} = \int d^4\theta \; [\hat{H}_1^{\dagger} e^{2gV + g'\hat{B}} \hat{H}_1 + \hat{H}_2^{\dagger} e^{2gV + g'\hat{B}} \hat{H}_2^{\dagger} \\
+ W\delta^2 \left(\bar{\theta} \right) + \bar{W} \, \delta^2 \left(\theta \right)]
\end{aligned}
\tag{4.3}
$$

here

g, g$'$– gauge coupling constants for SU(2) and U(1).

W$_\alpha$– SU(2) field strength $\equiv -\frac{1}{8}\overline{DD}e^{-2g\hat{V}} D_\alpha e^{2g\hat{V}}$

W$'_\alpha$– U(1) field strength $\equiv -\frac{1}{4} D\overline{D}_{\dot{\alpha}} D\hat{V}'$

here F$_{\mu\nu}$ and B$_\mu$ are contained in W$_\alpha$ and W$'_\alpha$.

4.3 The Superpotential

The Superpotential of the theory is defined by

W$\equiv W \left[\hat{L}, \hat{R}, \hat{Q}, \hat{u}, \hat{d}, \hat{H}_1, \hat{H}_2 \right]$

To guarantee a renormalizable theory the Superpotential must be

i) A function of $\hat{L}, \hat{R}, \hat{Q}, \hat{u}, \hat{d}, \hat{H}_1$and $\hat{H}_2$only and not there conjugate fields.

[1]See Table .3 for superfields which occur in the lagrangian below.

ii) At maximum a cubic in the superfields an analytic Function (derivative interactions are not allowed).

In MSSM the most general Superpotential takes on the form[21]

$$\begin{aligned} W &= \varepsilon_{ij}\mu\widehat{H}_1^i\widehat{H}_2^i + \varepsilon_{ij}\left[\lambda_L\widehat{H}_1^i\widehat{L}^j\widehat{R} + \lambda_d\widehat{H}_1^i\widehat{Q}^j\widehat{d} + \lambda_u\widehat{H}_2^i\widehat{Q}^j\widehat{u}\right] \\ &\quad +\varepsilon_{ij}\left[\lambda_1\widehat{L}^i\widehat{L}^j\widehat{R} + \lambda_2\widehat{L}^i\widehat{Q}^j\widehat{d}\right] + \lambda_3\widehat{u}\widehat{d}\widehat{d} \end{aligned} \tag{4.4}$$

here ε_{ij} is an anti-symmetric tensor defined as

$$\varepsilon_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

also λ_1,λ_2, λ_3 are all Yukawa coupling constants Here we have suppressed the generational indices on these constants. μ is a mass parameters required in order to get a physically acceptable theory. Because if theory has an additional symmetry (Peccei-Quinn symmetry) with $\mu = 0$ then spontaneous breakdown of this symmetry leads to an experimentally unacceptable Weinberg-Wilczek axion.

We now have all the L_{SUSY} terms. It can be checked that this Lagrangian is invariant under SU(3)$-$,SU(2)- U(1)-gauge and R symmetries apart its invariance under supersymmetric transformations. Now in the Lagrangian we will expand the chiral and vector superfields in terms of the component fields. e.g. the chiral field $\hat{L}\left(x,\theta,\overline{\theta}\right)$will have the following expansion,

$$\begin{aligned} \hat{L}\left(x,\theta,\overline{\theta}\right) &= \begin{pmatrix} \overline{\nu}_l\left(x,\theta,\overline{\theta}\right) \\ \widehat{l}\left(x,\theta,\overline{\theta}\right) \end{pmatrix} \\ &= \widetilde{L}\left(x\right) + i\theta\sigma^{\mu}\ \bar{\theta}\ \partial_{\mu}\widetilde{L}\left(x\right) - \frac{1}{4}\theta\theta\overline{\theta\theta}\partial^{\mu}\partial_{\mu}\widetilde{L}\left(x\right) \\ &\quad +\sqrt{2}\theta L^{(2)}\left(x\right) + \frac{i}{\sqrt{2}}\theta\theta\overline{\theta}\overline{\sigma}^{\mu}\partial_{\mu}L^{(2)}\left(x\right) + \theta\theta F_L\left(x\right) \end{aligned} \tag{4.5}$$

where

$$\hat{L}(x)=\begin{pmatrix}\overline{\nu}_l(x)\\ \widehat{l}(x)\end{pmatrix};\, L^{(2)}(x)=\begin{pmatrix}\nu_l^{(2)}(x)\\ l^{(2)}(x)\end{pmatrix}_L;\, F_L(x)=\begin{pmatrix}f^{\nu}(x)\\ f_L^l(x)\end{pmatrix}$$

For vector superfield, we use for convenience, Wess-Zumino gauge. In this gauge the expansions of SU(2) and U(1) gauge superfields are given as[]

$$\begin{aligned}\widehat{V}^a\left(x,\theta,\overline{\theta}\right) &= -\theta\sigma^\mu\,\bar{\theta}\,V_\mu^a(x)+i\theta\theta\overline{\theta}\overline{\lambda}^a(x)-i\theta\overline{\theta}\overline{\theta}\lambda^a(x)\\ &\quad+\frac{1}{2}\theta\theta\overline{\theta}\overline{\theta}D^a(x).\end{aligned}\tag{4.6}$$

Here $\lambda^a(x)$ is two component (Weyl) gaugino field, $V_\mu^a(x)$ is the superpartner of the SM gauge boson field and $D^{(a)}$ is the auxiliary field. If we expand all the fields in the Lagrangian we get several component fields. These are given in table (4).

Now with these expansions we obtain the off-shell Lagrangian i.e. the one containing the auxiliary fields. To get on- shell Lagrangian we will remove these fields through the Euler-Lagrange equations

$$\frac{\partial L}{\partial\Phi}-\partial_\mu\frac{\partial L}{\partial(\partial_\mu\Phi)}=0$$

where Φ is any Minkowski field. As formally auxiliary fields are defined as fields having no kinetic terms, so for these fields ,these Euler-Lagrange equations reduce to

$$\frac{\partial L}{\partial\Phi}=0.$$

Applying these simplified eqns. to various auxiliary fields we get the auxiliary fields in terms of $\hat{L}$, $\hat{R}$, $\hat{Q}$, $\hat{u}$, $\hat{d}$, $\hat{H}_1$and $\hat{H}_2$ .Thus substituting these results back in the Lagrangian we can get rid of auxiliary fields and can finally obtain the on-shell MSSM Lagrangian. Note that at this stage all of our particles are massless.

In analogy of SM we define

$$
\begin{aligned}
A_{\mu}(x) &= \cos\theta_w V'_{\mu} + \sin\theta_w, \\
V^3_{\mu} Z_{\mu}(x) &= cos\theta_w V^3_{\mu} - \sin\theta_w V'_{\mu},
\end{aligned}
$$

and

$$
W^{\pm}(x) = \frac{V^1_{\mu}(x) \mp V^2_{\mu}(x)}{\sqrt{2}},
$$

and using similar relations for spin $1/2$ gauginos involving λ^a and λ' we can define $\lambda_A(x)$, $\lambda_z(x)$, and $\lambda^{\pm}(x)$.The co-variant derivative now takes the form

$$
\begin{aligned}
D_{\mu} &= \partial_{\mu} + igT^a V^a_{\mu} + ig\frac{Y}{2} V'_{\mu} \\
&= \partial_{\mu} + \frac{ig}{\sqrt{2}} T^+ W^+_{\mu} + \frac{ig}{\sqrt{2}} T^- W^-_{\mu} + ieQA_{\mu} \\
&\quad + \frac{ig}{\cos\theta_w}\left[T^3 - Qsin\theta_w\right] Z_{\mu}
\end{aligned} \tag{4.7}
$$

where Q , the charge operator (with eigenvalues in units of elementary charge "e") is given interms of hyper charge Y and third. component of isospin,

$$
Q = T^3 + \frac{Y}{2}
$$

and

$$
T^{\pm} = T^1 \pm T^2
$$

here D_{μ} operates on SU(2) singlet. It is preferable to express the Lagrangian in four-component spinors notation for theoretical calculations. It can be easily achieved by introducing the Majorana spinors, having particle-antiparticle fields in one spinor. After having MSSM Lagrangian in 4-component spinor notation it is easy to compare it with SM Lagrangian. One can easily make the following observations ,

i) It contains the correct kinetic terms for the bosons and fermions of the theory.

ii) It contains the SM-interaction terms for SM particles and in addition terms between SM and SUSY particles and SUSY particles alone.

iii) We observe that for the Wino and charged Hhiggsino-fields, their charged conjugated fields also appear in the Lagrangian. This was not the case with SM. This new effect has the strange consequence that the theory will contain fermion-number violating vertices and propagators.

4.4 SUSY breaking

Upto now we have constructed a supersymmetric theory which is unbroken and particles and their partners are massless. It is typically assumed that SUSY breaking occurs at some high scale, say M_{planck} and perhaps results from some complete theory including gravity too. We have already discussed the spontaneous symmetry breaking of SUSY theories. Here for MSSM we look for the explicit symmetry breaking.

4.4.1 Explicit symmetry breaking of MSSM lagrangian

It has been shown that quadratic divergences still cancel even if, in the Lagrangian, we introduce [23]

i) Scalar mass terms - $m^2 |\phi_i|^2$

ii) Gaugino mass terms - $c_i\phi_i$

iii) Bilinear terms - $B_{ij}\phi_i\phi_j +$ h.c.

iv) Trilinear scalar interactions - $A_{ijk}\phi_i\phi_j\phi_k +$ h.c.

Where it should be kept in mind that

i) Linear terms are only gauge invariant for gauge singlet fields.

ii) We are not allowed to introduce additional masses to chiral fields

This will lead to soft breaking of SUSY. Thus the supersymmetric partners of the model are given masses by including explicit "soft" mass terms. Thus we have arbitrary masses of sparticles but this introduces a large number of unknown parameters (more than 50). It is true that remarkably, even with all these arbitrary parameters, the theory is still able to make some definitive predictions. This is of course, because the gauge interactions of the SUSY particles are completely fixed. But still there is a need for a theory in which SUSY soft breaking terms arise in order to reduce the parameter space.

Now we will examine how the electroweak symmetry is broken in this model.

4.4.2 Electro-Weak symmetry breaking

In SUSY-theories one has two kinds of potential Superpotential and scalar potentials one can find, from Superpotential both the scalar potential and the Yukawa interactions of the fermions with the scalars. However consider the scalar potential, which has its analogy in the scalar potential, which has its analogy in SM. Contributions to the MSSM scalar potential arise from

i) auxiliary field F and D.

ii) and soft scalar mass terms. i.e.

$$V_{MSSM} = V_D + V_F + V_{SOFT}.$$

But for symmetry breaking we will only consider the Higgs sector of the theory. The symmetry is broken when the neutral components of the Higgs doublets get vacuum expectation values,

and the negative minimum of the potential can be written as [22]

$$V_m = \frac{-1}{2\left(g^2 + g'^2\right)}\left[\left(m_1^2 - m_2^2\right) + \left(m_1^2 + m_2^2\right) Cos2\beta\right]$$

where $tan\beta = \frac{V_2}{V_1}$ and as v_1 , v_2 >0 so $0 \leq \beta \leq \frac{\pi}{2}$.

One important point here is that in the supersymmetric limit, where all (Soft) mass parameters of L_{SOFT} are set equal to zero, no electroweak breaking is possible. Thus gauge symmetry breaking is connected to the breaking of SUSY.

Before the symmetry was broken, the two complex Higgs doublets had 8 degrees of freedom. Three of these were absorbed to give the W and Z gauge bosons their masses, leaving 5 physical degrees of freedom. Now we have two charged Higgs bosons $\mathbf{H}^{\pm}$, three neutral Higgs bosons (one of which is CP-odd and 2 CP-even) **A**, **h** and **H.**

If we define two parameters

$(i)\, tan\beta = \frac{V_2}{V_1}$

$(ii)\, M_A =$ mass of pseudoscalar Higgs boson

Then masses of remaining Higgs can be calculated in terms of these parameters by convention **h** is taken to be the lighter of the neutral Higgs bosons. Then

$$M_{H^{\pm}}^2 = M_W^2 + M_A$$

It was predicted that is lighter than Z boson and so must be observable at LEP II. But since corrections to M_h^2 receive contributions from loops with both top-quark and s-quarks. If SUSY holds these contribution would cancel but as supersymmetry is broken we have to take the radiative corrections into account[22].

The MSSM prediction for W and Z mass is

$$\begin{aligned} M_{H^{\pm}}^2 &= \frac{1}{2} g^2 \left(v_1^2 + v_2^2\right) \\ M_Z^2 &= \frac{1}{2} \left(g^2 + g'^2\right) \left(v_1^2 + v_2^2\right) \end{aligned}$$

which are consistent with the results from the SM and are used to fix $v_1^2 + v_2^2$.

It is an important feature of MSSM that for large M_H the Higgs sector looks like that of the SM.

The leptons and quarks masses arise from Yukawa couplings of the theory, as in case of SM.

Charginos, the physical mass states $\widetilde{\chi}_1^{\pm}$; are linear combination formed from $\widetilde{W}^{\pm}$ and $\widetilde{h}^{\pm}$ i.e. winos and Higgsinos. By convention $M_{\widetilde{\chi}_1^{\pm}}$ is the lighter chargino. Similarly mixing of $\widetilde{b}$, $\widetilde{w}^3$ and neutral Higgsinos $\widetilde{h}_1^0, \widetilde{h}_2^0$ give physical states neutralino $\widetilde{\chi}_1^0$. By convention

$$M_{\widetilde{\chi}_1^0} < M_{\widetilde{\chi}_2^0} < M_{\widetilde{\chi}_3^0} < M_{\widetilde{\chi}_4^0}$$

The lightest neutralino, $\widetilde{\chi}_1^0$ is usually assumed to be the LSP.

4.5 R-parity

As we already discussed that a supersymmetric and gauge invariant Lagrangian may contain terms contributing to **L** and **B** voilation which can lead to proton decay. If the SUSY partners of the SM particles have masses on TeV scale, then these interactions are severely restricted by experimental measurements. To L and B interactions problem we have several approaches

i) Simply make the couplings small enough to provide experimental limits. This is allowed experimentally but does not look nice.

ii) To make either L interaction couplings or B couplings zero. This will forbid proton decay. But there is not much theoretical motivation for this approach.

What we actually want is that these terms be forbidden by certain symmetry so that they will not reappear at higher orders of perturbation theory. And this is what R-parity does. As we have already defined

$$R_p = (-1)^{2J+3B+L}$$

The R-parity conservation has profound experimental consequences.

i) SUSY partners can only be pair produced from SM particles.

ii) Production of SUSY particle will be followed by a cascade decay, until the lightest SUSY particle (LSP) is produced.

iii) When R-parity is conserved, LSP is absolutely stable.

iv) LSP behaves much like neutrino, when is tried to be detected as it is stable and neutral, and interacts weakly, only be exchange of a heavy virtual SUSY particle.

Thus with R-Parity conservation.

i) The generic signature of SUSY is events with missing energy

ii) LSP can be a candidate for dark matter

If R-parity is not conserved there will not be a stable LSP, and

$$LSP \rightarrow \text{ordinary} \quad \text{particles}$$

in this case missing energy will no longer be a signature for SUSY.

Chapter 5

SUPERSYMMETRIC GRAND UNIFIED THEORIES

A major motivation for introducing SUSY was the Hierarchy problem of GUTs. In SU(5) Grand Unified Theory spin 1/2 matter fields fit neatly into $\overline{\psi}(\ \overline{5})$ and $\chi(10)$ representations. While gauge bosons A_μ lie in (24) adjoint representation of SU(5).

Grand unified SU(5) is spontaneously broken to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ at a scale $M_x \approx 10^{14} GeV$ by a superheavy Higgs field $\phi(24)$. And then electroweak breaking occurs at scale $M_w = 10^2 GeV$ through Higgs multiplet H(5) .

To make this model supersymmetric we have to form chiral and gauge multiplets by introducing SUSY partners for all of the above particles. In addition we must include a second Higgs multiplet $H'(\bar{5})$ to give masses to both **u**-type and **d**-type quarks.

$$W(\phi^i) = a_i\phi^i + a_{ij}\phi^i\phi^j + a_{ijk}\phi^i\phi^j\phi^k$$

whereϕ^i are complex scalar bosons of gauge group.

The condition for SUSY to be broken at tree level, is that we require

$(i)\ \frac{\partial W}{\partial \phi^i} = 0$

(ii)-Hermitian and antihermitian ϕ commute.

If both these requirements have simultaneous solutions then SUSY is unbroken at tree level otherwise, SUSY is spontaneously broken. Now what we want is to break SU(5) to SU(3)$_C\times$ SU(2)$_L\times$ U(1)$_Y$ in such a way that supersymmetry remains unbroken (so that it is unbroken till weak scale). It is very simple to achieve this If we take the fields ϕ^iconsisting of a single traceless complex matrix transforming in the adjoint representation of SU(5) then in this case the most general choice of W is [24].

$$W = \frac{1}{2}MTr(A^2) + \frac{1}{3}\lambda TrA^3$$

The condition $\frac{\partial W}{\partial \phi^i} = 0$ gives three solution of A

$(i)\, A = 0$

$(ii)\, A = \frac{M}{3\lambda} diag\,(1,1,1,1,-4)$

$(iii)\, A = \frac{2M}{\lambda} diag\left(1,1,1,\frac{-3}{2},\frac{-3}{2}\right)$

corresponding to SU(5), SU(4) $\times$ U(1) and SU(3) $\times$SU(2) $\times$ U(1) unbroken gauge symmetries. As these solutions arise from condition for unbroken SUSY, they all have zero energy at tree level and so are completely degenerate[1]. This is illustrated schematically in Fig.

The 3rd solution is of our interest for which ϕ has a non zero **v.e.v.**

$$\langle 0\,|\phi|\,0\rangle = \text{diag}\ \ (2M, 2M, 2M, -3M, -3M)$$

which contributes to masses of H and H$'$ Higgs fields. The relevant part of the Superpotential is [8]

[1]A typical SUSY theory has many ground states, which are characterized by the expectation values of some scalar fields. All these states must be degenerate. This is similar to Spontaneous Symmetry Breaking but with SSB the degenerate vacua all have the same physics, while here they are physically inequivalent. They are not related to one another directly by any symmetry. The scalar fields which label these vacua are known as "moduli"

$$
\begin{aligned}
W &= \lambda' H'(\phi + m')H \\
&= \lambda'(2m + m')H'_{1,2,3}H_{1,2,3} + \lambda'(-3m + m')H'_{4,5}H_{4,5}
\end{aligned}
$$

It is the later term which is responsible for SM electroweak breaking at scale M_W. In order that the doublet components (4,5) have mass $\mathbf{0}(M_W)$ rather than $\mathbf{0}(M_X)$, we require,

$$M' = 3M$$

where M and M$'$ are of $\mathbf{0}(M_W)$. This implies a fine tuning, which is just the **HIERARCHY** Problem of GUTs again. Thus it seems that we have not achieved anything but infect we do because of the lack of renormalization of the mass parameters in SUSY theory, the fine tuning is required just once in the original Lagrangian and not in each order of perturbation theory separately. Now let us very briefly review SGUT models.

5.1 SUSY GUTs

In a standard SUSY GUT model we assume that $SU(3)_C \times SU(2)_L \times U(1)_C$ gauge coupling constants are unified at a high scale $M_x \approx 10^{16} GeV$ [21]

$$\sqrt{\frac{5}{3}} g_1(M_x) = g_2(M_x) = g_3(M_x) \equiv g_x$$

The gaugino masses are also assumed to unify.

$$M_i(M_x) \equiv m_{1/2}$$

At lower energies the gaugino masses have scale in the same way as the corresponding coupling constant i.e.

$$\frac{M_i(M_W)}{m_{1/2}} = \frac{g_i^2(M_W)}{g_x^2}$$

which implies,

$$\begin{aligned} M_1 &= \frac{5}{3}\tan^2\theta_w M_2 \\ M_2 &= \frac{\alpha}{\sin^2\theta_w}\frac{1}{\alpha_s}M_3 \end{aligned}$$

This shows that the gluino mass (M_3) is always heaviest of the gaugino masses. This is one of the major predictions of SUSY GUT.

Another important feature of SUSY GUTs is that it can possibly give answer of the question that ,

$$\text{why} \;\; \mu^2 < 0 \;\; ?$$

which is the basic requirement for SSB at electroweak scale in SM. In a typical SUSY GUT model it can be seen that the lightest Higgs boson mass becomes negative around the electroweak scale [25] provided M_{TOP} is large. In fact, this mechanism only work for $M_{TOP} \sim 175$ GeV.

And it is also worth noting that SUSY GUTS can naturally accommodate a large top quark mass [21]

$$M_{TOP} \sim (200GeV)\, tan\beta$$

and for $\tan\beta \sim 2$, M_{TOP} is close to experimental value.

5.2 Unification of running coupling constants

In a gauge theory, coupling constants scale with energy according to the β-function. Hence having measured a coupling constant at one energy scale, its value at any other energy can be predicted.

At one loop

$$\frac{1}{\alpha_i\left(\mu^2\right)} = \frac{1}{\alpha_i\left(\mu^2\right)} + \frac{b_i}{2\pi}\log\left(\frac{Q}{\mu}\right)$$

As in SUSY GUTs we have more particles in low energy sector, the evaluation of coupling constants is changed from those we had in case of non- supersymmetric model $(ch.2)$.Now they are given by

$$\begin{aligned} b_1 &= -2N_g - \frac{3}{10}N_H \\ b_2 &= 6 - 2N_g + \frac{1}{2}N_H \\ b_3 &= 9 - 2N_g \end{aligned}$$

Where N_g is number of generations (up to scale $\sim M$) and N_H is number of Higgs doublets, which is 2 in case of SUSY GUT. fig. shows a low energy SUSY model. From experimentally measured values at Z-pole. The SUSY thresholds are taken to be at **1** TeV. We see that coupling constants meet at a scale $\approx 10^{16}GeV$. [26, 27, 28] with $\alpha_i \approx 1/25$.

The observation that the measured coupling constants tend to meet at a point when evaluated to high energy,assuming the β-function of a low energy SUSY model has led to wide spread acceptance of a standard SUSY GUT model.

Chapter 6

SUPERGRAVITY

Up to now we have taken SUSY as a global symmetry. What about its local version? That is when the parameters of SUSY transformation become a function of space time.

$$\varepsilon = \varepsilon(x).$$

From our previous experience in field theory we know that invariance under any local symmetry requires the introduction of new vector fields in the theory. Consider the free spin 3/2 massless Rarita- Schwinger eqn.[29] :

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\psi_\sigma = 0. \tag{6.1}$$

This equation is gauge invariant and therefore requires local transformation:

$$\delta\psi_\mu(x) = \partial_\mu\epsilon(x). \tag{6.2}$$

It is an interesting fact that the above eqn. and the coupling of the spin 3/2 field to a conserved current can be derived, conversly, by the requirement of local supersymmetry on a supersymmetric lagrangian.

Thus requirement for local SUSY leads to the introduction of a Majorana field with spin 3/2 . But this introduction of sopin 3/2 field gives rise to another variation in the lagrangian as an additional term:

$$\delta L = ik\epsilon\gamma_\nu\psi_\mu\theta^{\nu\mu}, \tag{6.3}$$

and this new term can be cancelled only if we include the gravitational coupling i.e. to keep the theory supersymmetric we require a massless spin 2 field which can be identified with the Graviton field . Thus giving a hint of unification of gravity with other three forces of nature[30, 31, 32], a step towards theory of everything (TOE) ? [1]

From dimensional arguments it follows that the coupling constant k must have dimensions of M^{-1} i.e, local supersymmetry requires a dimensional coupling, a fact contrary to our observation so far. On dimensional grounds we can define

$$k = \sqrt{8\pi G_N}$$

where G_N is the gravitational constant with dim. M^{-2}. k is used as the universal gauge coupling associated with local supersymmetry.

It is very difficult to construct globally supersymmetric models where SUSY is spontaneously broken at the weak scale. This problem can be solved if we assume that SUSY is broken in a "hidden" sector at a scale $\mu \gg M_w$. It is also assumed that this sector then interacts with "observable" sector only gravitationally . This led to the development of supergravity GUT models..Because of the assumed symmetries, various soft SUSY breaking parameters become related and thus the parametrization of low energy effective Lagrangian needs just a few parameters.

The other assumptions made in SUSY GUTs are,

i) All scalars have same mass, m_o.

[1] In a non technical way this can be understood by noting the fact that when spin of a particle is changed , not only its internal properties are changed but also the space-time properties.ie. particle is "displaced" also. And this is exactly what a gauge transformation in the theory of gravity does. Consequently a super-gauge theory can only be formulated if it is combined with gravity i.e supergravity.

ii) All gauginos have same mass, $m_{1/2}$

iii)All trilinear couplings are same, A_0.

These assumptions have the tremendous advantage that the SUSY sector is fully described by just **5** input parameters at the GUT scale [33, 34] that are,

m_o , $m_{1/2}$, , A_0 , tan β , sign (μ)

First 3 parameters are specified at the unificatioin scale, while tan β is taken at the scale Mz. This set of assumption is often called the "superstring inspired " SUSY GUT" or **mSUGRA** [2]or the constrained MSSM (CMSSM). Thus predictions for various SUSY particle masses (as a function of m_o , $m_{1/2}$) can be found for given values of A_0 , tan β and sign (μ) . Actually these parameters completely determine all the sparticle masses and couplings.

The **mSUGRA** model provides a consistent framework for SUSY phenomenology. An interesting aspect of mSUGRA is that with R-parity invariance the lightest neutralino is the LSP and thus a candidate for Cold dark matter (CDM) . Thus it also provides a natural candidate for **CDM.**

In the mSUGRA scenario the resulting low energy spectrum tends to have the following properties.

i) the weak-scale gaugino mass parameters have the same ratio as that of the relevant coupling

$$M_1 : M_2 : M_3 \sim \alpha_1 : \alpha_2 : \alpha_3$$

ii) The μ parameter tends to be large i.e. $|\mu| \gg M_2$, implying that lightest neutralino $\widetilde{\chi}_1^0$ is primarily bino i.e. corresponds to $\widetilde{B}$, and $\widetilde{\chi}_2^0$ and lightest chargino are wino i.e. $\widetilde{\chi}_1^0, \widetilde{\chi}_1^0$ correspond to $\widetilde{W}$. It also implies that heavier neutralinos ($\widetilde{\chi}_{3,4}^0$) and chargino ($\widetilde{\chi}_2^{\pm}$) are primarily Higgsino.

ii) If $M_0 \gg M_{1/2}$ the squarks and sleptons are nearly degenerate and heavier than

[2]minimal supergravity

$\tilde{\chi}^0_{1,2}$ and $\tilde{\chi}^{\pm}_1$.

iii) If $M_0 \leq M_{1/2}$, quarks are heavier than sleptons. These properties will be useful while looking for the SUSY signature.

N = 1 supergravity leaves many questions unanswered including,

iv) Matter fields are chosen to fit phenomenology and are not a natural out-come of the theory.

iii) Here not only problem of divergences rises again but is also more severe than global SUSY or non-supersymmetric theories.

In order to find answers to these and many other questions it would be interesting to go beyond N=1 SUSY.

The most interesting of these is the N=8 supergravity. Here every super-multiplet should have all spin values from 0 to 2. In a more understandable way, the spin can change from -2 to +2 in eight steps of one half unit. But there must be only one graviton as in Einstein's theory only one type of gravitational force is allowed and therefore we can only have one N=8 supermultiplet ,thus not allowing any extension or addition . This single supermultiplet contains so many particles that all existing particles can be taken care of. Thus this theory seems to have features of a theory of every thing but there is a problem that it is a **nonrenormalizable** . We surely have to think of something else and that "something else" is to try all those theories in spaces with more dimensions than our 4-dimensional space-time. This idea was first propsed by Kaluza and was further elaborated by Klein. The 11 dimensional supergravity was found to be much more symmetric and accommodating all the particles. Here 7 out of 11 dimensions are assumed to be rolled up leaving 4 physical space-time dimensions. Could this be a theory of every thing? Unfortunately (or may be fortunately) in this theory it is not possible to have infinities cancelled in diagrams with more than 7 loops .

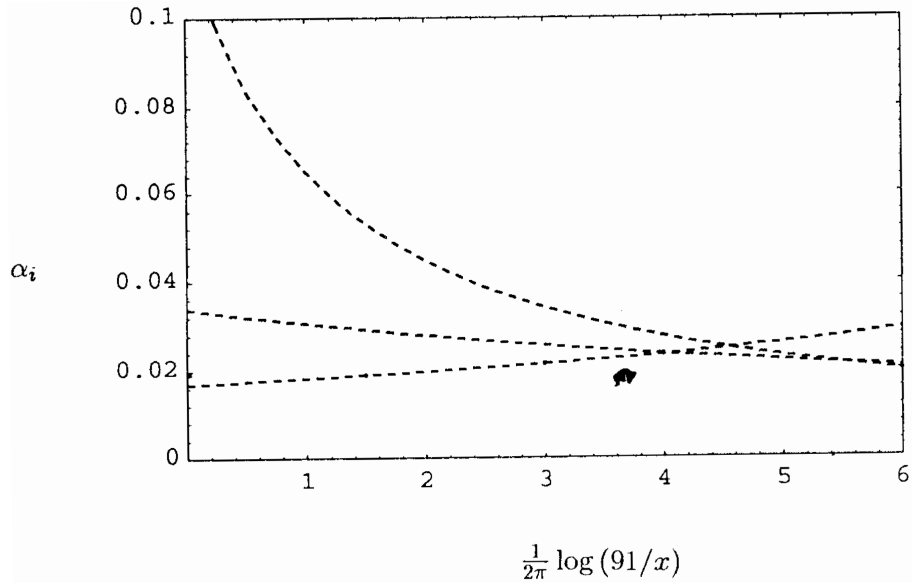
So, is there nothing more which supersymmetry, with all its beauty and completeness, can do ? the answer may lie in **Superstrings.**

Superstring theory , as a candidate theory of elementry particle physics at the Planck scale was first propsed in 1974. The 10- dimensional super Yang Mills theories emerge as a limiting case of superstring theories.

It was seen that the massless closed-string (which are bound states of open strings) spectrum contain a massless gravitino field . The only way such a field can have consistent couplings is if the theory has local supersymmetry for which gravitino is the gauge field. This requires that the entire spectrum falls in to supermultiplets, showing the presence of SUSY as an underlying theory in superstrings. In fact the set of massless states consists of a vector and a Majorana-Weyl spinor , which in a limiting case, is described by a supersymmetric Yang Mills theory in 10-dimensions.. By dimensional reduction , one obtains the N=4 Yang Mills theory in 4-dim. and representations are the same as that of an 11-dimension Supergravity (SUGRA).

Comments and Conclusion

In this dissertation, we have reviewed the basic theory of supersymmetry with a special emphasis on supersymmetric Standard Model, supersymmetric grand unified theory and supersymmetric gravity without going into extensive technical detail. We have highlighted the cancellation of divergences , economisation of SM parameters, R-parity phenomenological consequences, unification of gauge couplings, etc. in the context of SUSY theories. We find that the SUSY theories afford a most attractive and complete framework leading to superstrings which is a strong candidate for theory of everything. The fate of SUSY will be finalized in next few years with the run of LHC, Tevatron, LEP II etc., which we are all looking forward to, with our fingers crossed.



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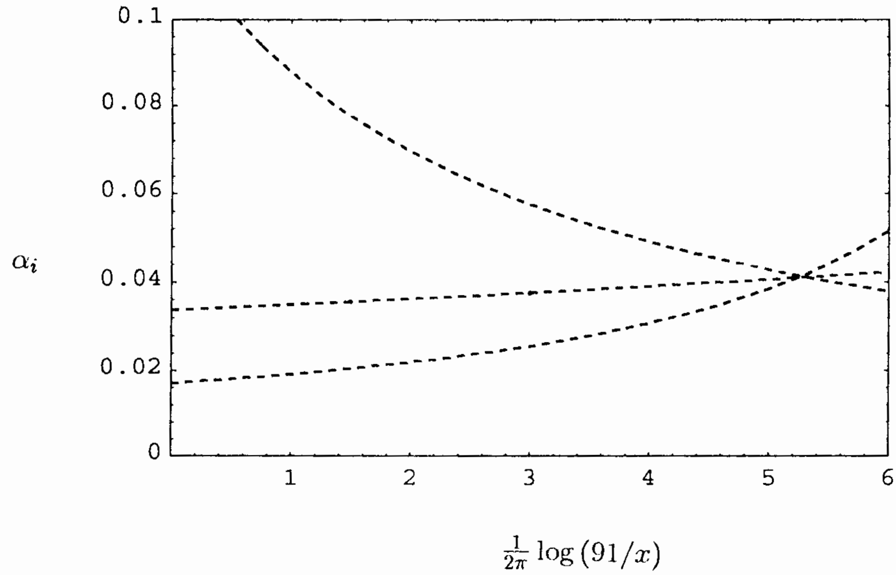
$\frac{1}{2\pi}\log(91/x)$

*Fig.*1



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$\frac{1}{2\pi}\log(91/x)$

*Fig.*2

Table - 1

Fields	Component Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Matter (Fermions) Fields	$L_{L\equiv} \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$	$(1, 2, -1)$
	e'^{-}_R	$(1, 1, -2)$
	$q_{L\equiv} \begin{pmatrix} u \\ d \end{pmatrix}_L$	$(3, 2, 1/3)$
	u'^c_L	$(\overline{3}, 1, -4/3)$
	d'^c_l	$(\overline{3}, 1, 2/3)$
HiggsField	Φ	$(1, 2, 1)$
Guage Fields	w	$(1, 3, 0)$
	B	$(1, 1, 0)$
	g	$(8, 1, 0)$

Table - 2

Quantity	Data(Aug'97)	SM Fit	Pull$_{(\equiv \frac{D-F}{Error})}$
$m_{z(Gev0}$	91.1867(20)	91.1866	0.0
$\Gamma_{z(Gev)}$	2.4948(25)	2.4966	-0.7
$\sigma_{h(nb)}$	41.486(53)	41.467	0.4
R_h	20.775(27)	20.756	0.7
R_b	0.2170(9)	0.2158	1.4
R_c	0.1734(48)	0.1723	-0.1
A^l_{FB}	0.0171(10)	0.0162	0.9
A_τ	0.1411(64)	0.1470	-0.9
....	....	....	......
$m_{w(Gev)}$	80.43(8)	80.375	0.7
....	....	....	......
$m_{top(Gev)}$	175.6(5.5)	173.1	0.4

Table - 3

Super fields	Comp. fields	*Name*	U(3), SU(2), U(1)
Matter Fields			
$\hat{Q}\left(x,\theta,\overline{\theta}\right) = \begin{matrix} \hat{u}_l\left(x,\theta,\overline{\theta}\right) \\ \hat{d}_l\left(x,\theta,\overline{\theta}\right) \end{matrix}$	u_L d_l $\tilde{u}_L$ $\tilde{d}_l$	Quarks Squarks	$\left(3,2,\frac{1}{3}\right)$
$\hat{u}\left(x,\theta,\overline{\theta}\right) = \hat{u}_L^c\left(x,\theta,\overline{\theta}\right)$	u_L^c $\tilde{u}_L^c$	R.Handed Upquarks& Upsquarks	$\left(3^*,1,-\frac{4}{3}\right)$
$\hat{d}\left(x,\theta,\overline{\theta}\right) = \hat{d}_L^c\left(x,\theta,\overline{\theta}\right)$	d_L^c $\tilde{d}_L^c$	R.Handed dquarks& dsquarks	$\left(3^*,1,\frac{2}{3}\right)$
$\hat{L}\left(x,\theta,\overline{\theta}\right) = \begin{pmatrix} \overline{\nu}_l\left(x,\theta,\overline{\theta}\right) \\ \hat{l}\left(x,\theta,\overline{\theta}\right) \end{pmatrix}$	ν_L e_L^- ν_L e_L^-	Leptons Sleptons	$1(,2,-1)$
$\hat{R} = \hat{e}_l^c\left(x,\theta,\overline{\theta}\right)$	e_L^c $\tilde{e}_L^c$	Antilepton Antislepton	$(1,1,2)$
Gauge fields			
$\hat{V}^a\left(x,\theta,\overline{\theta}\right)$	$\begin{pmatrix} W^\pm \\ W^3 \end{pmatrix}$ $\begin{pmatrix} \widetilde{W}^\pm \\ \widetilde{W}^3 \end{pmatrix}$	Gauge bosons Gaugino	$(1,3,0)$
$\hat{V}^a\left(x,\theta,\overline{\theta}\right)$	B $\tilde{B}$	Gauge boson *Gaugino*	$(1,1,0)$

Table - 3 (continued)

Super fields	Comp. fields	Name	U(3), SU(2), U(1)
Higgs fields			
$\widehat{H}_1\left(x,\theta,\overline{\theta}\right) = \begin{matrix} \widehat{H}_1^1\left(x,\theta,\overline{\theta}\right) \\ \widehat{H}_1^2\left(x,\theta,\overline{\theta}\right) \end{matrix}$	Φ_u^+ Φ_u^0 $\widetilde{\Phi}_u^+$ $\widetilde{\Phi}_u^0$	Higgs fields Higgsino	$(1,2,+1)$
$\widehat{H}_2\left(x,\theta,\overline{\theta}\right) = \begin{matrix} \widehat{H}_2^1\left(x,\theta,\overline{\theta}\right) \\ \widehat{H}_2^2\left(x,\theta,\overline{\theta}\right) \end{matrix}$	Φ_d^0 Φ_d^- $\widetilde{\Phi}_d^0$ $\widetilde{\Phi}_d^-$	Higgs fields Higgsino	$(1,2,-1)$

Table - 4

Name		Symbol	Spin	Charge
Leptons	ν_L	$L^{(2)^1}$	1/2	0
	e_L^-	$L^{(2)^2}$	1/2	-1
	e_L^c	$R^{(2)}$	1/2	1
SLeptons	$\tilde{\nu}_L$	$\tilde{L}^1$	0	0
	$\tilde{e}_L^-$	$\tilde{L}^2$	0	-1
	$\tilde{e}_L^c$	$\tilde{R}$	0	1
Quarks	u_L	$Q^{(2)^1}$	1/2	2/3
	d_l	$Q^{(2)2}$	1/2	-1/3
	u_L^c	u	1/2	-2/3
	d_L^c	d	1/2	1/3
Squarks	$\tilde{u}_L$	$\tilde{Q}^{(2)^1}$	0	2/3
	$\tilde{d}_l$	$\tilde{Q}^{(2)2}$	0	-1/3
	$\tilde{u}_L^c$	$\tilde{u}$	0	-2/3
	$\tilde{d}_L^c$	$\tilde{d}$	0	1/3
Higgs Bsons		H_1^1	0	0
		H_1^2	0	-1
		H_2^1	0	1
		H_2^2	0	0
Higgsino		$\Psi_{H_1}^1$	1/2	0
		$\Psi_{H_1}^2$	1/2	-1
		Ψ_{H2}^1	1/2	1
		Ψ_{H2}^2	1/2	0

Table - 4 (continued)

Name	Symbol	Spin	Charge
Gauge bosons	V_μ^a	1	-
	V_μ'	1	-
Gaugino	λ^a	1/2	-
	λ'	1/2	-
Leptons	f^u	0	0
	f_L^l	0	-1
	f_R^l	0	1
Quarks	f_l^u	0	2/3
	f_l^d	0	-1/3
	f_R^u	0	-2/3
	f_R^d	0	1/3
Higgs	f_1^1	0	0
	f_1^2	0	-1
	f_2^1	0	1
	f_2^2	0	0
Gauge	D^a	1	-
	D'	1	-

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